

Final Report

**Bilateral inter-comparison in the gas flow range from
0.5 m³/h to 75 m³/h with sonic nozzles**

EURAMET Project No. 1783



Flow

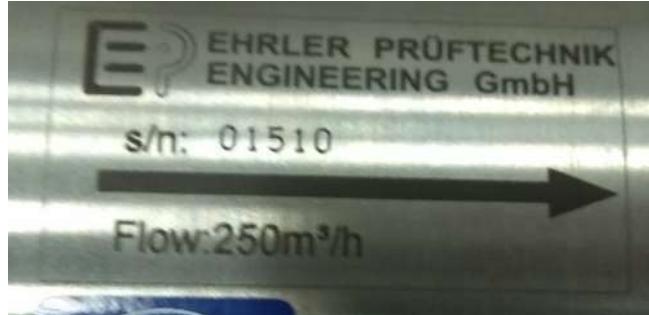
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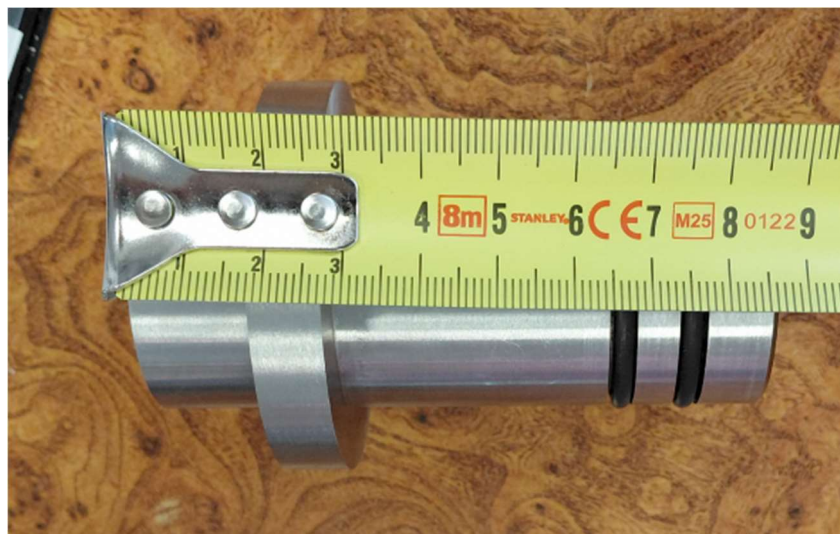
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2.1. Sonic nozzle 75 m³/h



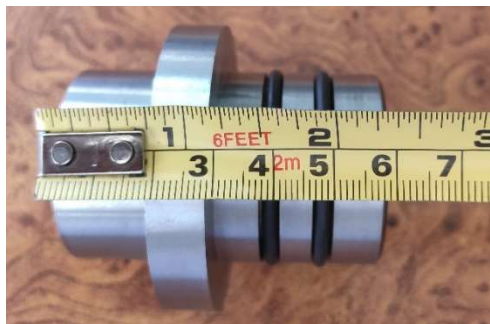
2.2. Sonic nozzle 64 m³/h



2.3. Sonic nozzle 32 m³/h



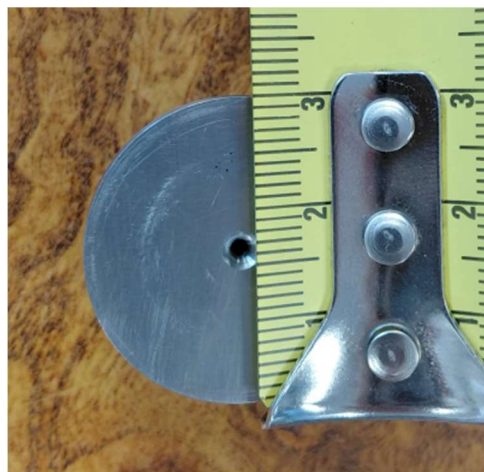
2.4. Sonic nozzle 16 m³/h



2.5. Sonic nozzle 8 m³/h



2.6. Sonic nozzle 1 m³/h



2.7. Sonic nozzle 0.5 m³/h



3. Calibration procedure

The calibration test procedure is mentioned in the document *Wendt, G.; Dietrich, H.; Jarosch, B.; Joest, R.; Natz, B.; Frössl, F.; Ruwe, M.: PTB testing instruction Volume 25: Gas meters – Test rigs with critical nozzles (English version 2000: 91 pages)*.

The calibrations of a sonic nozzles with nominal flow rate **75 m³/h** and **64 m³/h** were performed with air according to the chapter 3.2.1 *Determination of nozzle reference value $Q_{v,20,dry,Air}$ (one point test)*.

The calibrations of sonic nozzles with nominal flow rates **32 m³/h, 16 m³/h, 8 m³/h, 1 m³/h and 0.5 m³/h** were performed according to the chapter 3.2.2 *Determination of nozzle reference value $Q_{v,20,tr,1000}$ (two points test)*.

The ambient temperature in laboratory had to be $(21 \pm 1) ^\circ\text{C}$ and the relative humidity in laboratory had to be less than 80 % during the tests.

4. Test facility and obtained results

4.1. Slovak Republic

The primary gas flow standard of the **Bell Prover** type was established at the Slovak Institute of Metrology (SMU) between 1999 and 2002. Until 2022, the flow rate range of the standard was defined within the interval (1 to 65) m³/h, with an expanded measurement uncertainty of 0.12% ($k = 2$).

During the revision of the CMC tables performed in 2022, the flow rate range of the standard was updated to (0.5 to 75) m³/h. At the same time, the expanded measurement uncertainty was revised to 0.13% ($k = 2$) for the flow range $(0.5 \leq Q < 1)$ m³/h and to 0.09% ($k = 2$) for the flow range $(1 \leq Q \leq 75)$ m³/h.

This primary standard is listed in the CMC database maintained by the *International Bureau of Weights and Measures* (BIPM) under the designation SMU (SK 13) with KCDB CMC ID EURAMET-M-SK-0000063C-3.

Range of flow rate: (0.5 to 75) m³/h

Temperature: $(20 \pm 2) ^\circ\text{C}$

Working pressure: atmospheric conditions

Uncertainty CMC ($k=2$): 0.13 % for the flow rate range $(0.5 \leq Q < 1)$ m³/h

0.09 % for the flow rate range $(1 \leq Q \leq 75)$ m³/h

NMI Service Identifier: SMU (SK 13)

KCDB CMC ID: EURAMET-M-SK-0000063C-3

Place of calibration: Slovak Institute of Metrology
Karloveská 63, 842 55 Bratislava, Slovak Republic

The standard operates on a **volumetric principle of gas flow realization**. The main component of the facility is a bell with a precisely determined internal surface immersed in a sealing liquid (oil with low viscosity and low evaporation rate under normal pressure and temperature conditions). The movement of the bell displaces a precisely defined volume of air, which is measured using connected measuring instruments. The generated gas volume is traceable to the SI base units, specifically the units of length and time. The picture of Bell Prover in SMU (*Figure 1*) is mentioned down.



Figure 1 Bell Prover in SMU

Results:

Nozzle-ID s.n.	$Q_{V,20,dryAir}$	$U(k=2)$	$U(k=2)$ in CMC in KCDB
	[m ³ /h]	[%]	[%]
01508	74.52313	0.09	0.09
JT-01-64000-1999	63.97613	0.09	0.09

Nozzle-ID s.n.	$Q_{v,20.tr.1000}$	c_{pE}	$U(k=2)$	$U(k=2)$ in CMC in KCDB
	[m ³ /h]	[1/mbar]	[%]	[%]
JT-01-32000-1999	32.04876	$1.28 \cdot 10^{-5}$	0.09	0.09
JT-01-16000-1999	15.93723	$7.49 \cdot 10^{-5}$	0.09	0.09
JT-03-8000-1999	8.10356	$1.09 \cdot 10^{-5}$	0.09	0.09
JT-02-1000-1998	1.01768	$3.29 \cdot 10^{-5}$	0.09	0.09
JT-02-500-1999	0.49224	$2.48 \cdot 10^{-5}$	0.13	0.13

4.2. Czech Republic

Place of the test

Czech Metrology Institute, Gas Flow Department, Prumyslova 455, 530 03 Pardubice, Czech Republic

The test facility

A new national standard Bell Prover with the range from 0.5 m³/h to 280 m³/h was used for the calibrations of all the sonic nozzles. The bell was dimensionally very accurately evaluated by PTB. The manufacturer was company EP Ehrler Prüftechnik Engineering GmbH, Germany. The Bell Prover consists of:

- exactly dimensioned stainless steel bell
- connection system with switching device
- oil Shell Morlina 5
- fan, vacuum pump
- pressure vessel 2.7 m³
- control PC with software
- electronic digital thermometers with 0.01°C graduation scale, 4 pieces of manufacturer Temperaturmeßtechnik Geraberg GmbH,
- electronic digital pressure instruments with 1 Pa graduation scale, 5 pieces
 - manufacturer PAROSCIENTIFIC, INC, 1 piece
 - manufacturer YOKOGAWA, 3 pieces
 - manufacturer ROSEMOUNT, 1 piece
- incremental rulers with 0.001 mm graduation scale, 2 pcs
 - producer HEDENHEIN
- timing circuit in a collecting unit serving as a stopwatch with a message of 0.001 s, 1 piece
 - manufacturer Brehm + Jung
- hygrometer, 1 pc
 - manufacturer JUMO

The nozzles were tested in sinking mode. Waiting time between measurements is 300 seconds. This Bell Prover is mentioned in CMC with NMI Service Identifier CZ21 and $U(k=2)=0.07\%$. The picture of Bell Prover in CMI (*Figure 2*) is mentioned down:



Figure 2 Bell Prover in CMI

Results:

Nozzle-ID s.n.	Nominal flow rate	$Q_{v,20,dryAir}$	U(k=2)	U(k=2) in CMC in KCDB
	[m ³ /h]	[m ³ /h]	[%]	[%]
01508	75	74.55176	0.072	0.07
JT-01-64000-1999	64	63.95706	0.072	0.07

Nozzle-ID s.n.	Nominal flow rate	$Q_{v,20,tr,1000}$	c_{pE}	U(k=2)	U(k=2) in CMC in KCDB
	[m ³ /h]	[m ³ /h]	[1/mbar]	[%]	[%]
JT-01-32000-1999	32	32.03382	$9.78 \cdot 10^{-6}$	0.071	0.07
JT-01-16000-1999	16	15.93662	$1.26 \cdot 10^{-6}$	0.071	0.07
JT-03-8000-1999	8	8.10456	$6.45 \cdot 10^{-6}$	0.072	0.07
JT-02-1000-1998	1	1.01818	$9.02 \cdot 10^{-6}$	0.072	0.07
JT-02-500-1999	0.5	0.49270	$1.04 \cdot 10^{-5}$	0.077	0.07

5. Determination of the reference values in determined flow rates

The reference value was determined in each flow rate separately, it means separately for each sonic nozzle. The method of determination of the reference value in each flow rate corresponds to the procedure A presented by M.G.Cox²⁾. Results from independent laboratories were taken into account for the determination of the key comparison reference value (KCRV) and of the uncertainty of the key comparison reference value.

5.1. The determination of the Key Comparison Reference Value (KCRV) and its uncertainty

The reference value y was be calculated as weighted mean of parameters (determined flow rates) $Q_{v,20,tr}$ or $Q_{v,20,tr,1000}$.

²⁾ Cox M.G., *Evaluation of key comparison data*, Metrologia, 2002, **39**, 589-595

$$y = \frac{\frac{x_1}{u_{x1}^2} + \frac{x_2}{u_{x2}^2}}{\frac{1}{u_{x1}^2} + \frac{1}{u_{x2}^2}}, \quad [1]$$

where x_1, x_2, x_{n3} are parameters $Q_{v,20,tr}$ or $Q_{v,20,tr,1000}$ of a sonic nozzle in different independent laboratories 1,2,3 [m³/h]

u_{x1}, u_{x2}, u_{x3} are standard uncertainties (not expanded) in different independent laboratories 1,2 [m³/h]

The standard uncertainty of the reference value u_y is given by

$$\frac{1}{u_y^2} = \frac{1}{u_{x1}^2} + \frac{1}{u_{x2}^2} \quad [2]$$

The expanded uncertainty of the reference value $U(y)$ is

$$U(y) = 2 \cdot u_y \quad [3]$$

The chi-squared test for consistency check will be performed using parameters $Q_{v,20,tr}$ or $Q_{v,20,tr,1000}$ of a sonic nozzle. At first the chi-squared value χ_{obs}^2 will be calculated by

$$\chi_{obs}^2 = \frac{(x_1 - y)^2}{u_{x1}^2} + \frac{(x_2 - y)^2}{u_{x2}^2} \quad [4]$$

The degrees of freedom ν will be assigned

$$\nu = n - 1 \quad [5]$$

where n is number of evaluated laboratories.

The consistency check will be failing if

$$Pr\{\chi_\nu^2 > \chi_{obs}^2\} < 0,05 \quad [6]$$

(The function $CHIINV(0,05; \nu)$ in MS Excel will be used. The consistency check will be failing if $CHIINV(0,05; \nu) < \chi_{obs}^2$)

If the consistency check does not fail then y will be accepted as the key comparison reference value x_{ref} and $U(y)$ will be accepted as the expanded uncertainty of the key comparison reference value $U(x_{ref})$.

Theoretically if the consistency check fails then the laboratory with the highest value of $\frac{(x_i - y)^2}{u_{xi}^2}$ is excluded for the next round of evaluation and the new reference value y (WME), the new standard uncertainty of the reference value u_y and the chi-squared value χ_{obs}^2 is calculated again without the values of excluded laboratory. Hence the consistency check is calculated again. This procedure can be repeated till the consistency check will pass.

According to this paper³⁾ the coefficient P_i was also calculated according to the equation:

$$P_i = \Phi \left(\frac{x_i + 1.96u_{lab,i} - x_{KCRV}}{u_{KCRV}} \right) - \Phi \left(\frac{x_i - 1.96u_{lab,i} - x_{KCRV}}{u_{KCRV}} \right) \quad [7]$$

Φ is the cumulative distribution function of the normal distribution. This formula was used in MS Excel:

$$P_i = \text{NORM.DIST}((X_i + 1.96 * u_{lab} - X_{KCRV}) / u_{KCRV}; 0; 1; \text{TRUE}) - \text{NORM.DIST}((X_i - 1.96 * u_{lab} - X_{KCRV}) / u_{KCRV}; 0; 1; \text{TRUE}) \quad [8]$$

Data can be considered consistent in a comparative analysis only if the condition $P_i > 0.35$ is fulfilled.

5.2. The determination of the differences “Lab to KCRV” and “Lab to Lab” as well as their uncertainties and Degrees of Equivalence

When the KCRV was determined, the differences between the participating laboratories and the KCRV were calculated according to

$$d_i = x_i - x_{ref} \quad [8]$$

Based on these differences, the Degree of Equivalence (DoE) was calculated according to:

$$E_i = \frac{d_i}{U(d_i)} \quad [9]$$

The DoE is a measure for the equivalence of the results of any laboratory with the KCRV or with any other laboratory, respectively:

- The results of a laboratory is **equivalent (passed)** if $|E_i|$ or $|E_{ij}| \leq 1$.
- The laboratory was determined as **not equivalent (failed)** if $|E_i|$ or $|E_{ij}| > 1.2$.
- For values of DoE in the range $1 < |E_i|$ or $|E_{ij}| \leq 1.2$ we define “**warning level**” were actions to check is recommended to the laboratory.

The reason for such “warning level” is that we have to consider the confidence in the determination of the uncertainties (for the results of labs as well the KCRV). Conventionally we work at a 95%

³⁾ Enrico Frahm and John Wright, *Evaluating Inter-laboratory Comparison Data*, in *Proceedings of FLOMEKO 2022*, Chongqing, China, 2022.

confidence level. Therefore in some comparisons a range up to $|E| < 1.5$ is used for these “warnings”⁴⁾. This is a reasonable value where stochastic influences dominate the uncertainty budgets. In the case of comparisons for gas flow, the smaller value 1.2 was chosen, which reflects the dominance of non-stochastic parts of uncertainty compared to the stochastic parts. (The reproducibility is usually much better than the total uncertainty of a laboratory).⁵⁾

It is possible find the different cases:

Differences to the KCRV

A) Independent laboratories with contribution to the KCRV

The covariance between the result of a laboratory (with contribution to the KCRV) and the KCRV is the variance of the KCRV itself.⁶⁾

$$\Rightarrow u(di) = \sqrt{u_{xi}^2 + u_{xref}^2 - 2u_{xref}^2} = \sqrt{u_{xi}^2 - u_{xref}^2} \quad [10]$$

B) Independent laboratories without contribution to the KCRV

There is no covariance between the result of a laboratory without contribution and the KCRV.

$$\Rightarrow u(di) = \sqrt{u_{xi}^2 + u_{xref}^2} \quad [11]$$

6. Results

6.1. Czech Republic

Nominal flow rate	$Q_{V,20,dryAir}$	Uf	$1/u^2$	wi	chi	Pi	Data consistency
[m ³ /h]	[m ³ /h]	[m ³ /h]					
75	74.5518	0.0537	1388.2869	0.6096	0.1734	0.97	OK
64	63.9571	0.0460	1886.3317	0.6099	0.1044	0.98	OK
32	32.0338	0.0227	7732.6062	0.6166	0.2538	0.97	OK
16	15.9366	0.0113	31242.8650	0.6164	0.0017	0.99	OK
8	8.1046	0.0058	117472.4098	0.6097	0.0180	0.99	OK
1	1.0182	0.0008	7046028.8412	0.5964	0.2910	0.97	OK
0.5	0.4927	0.0004	27791600.338	0.7399	0.3976	0.92	OK

⁴⁾ C. Ullner et al., *Special features in proficiency tests of mechanical testing laboratories*, and P. Robouch et al., *The „Naji Plot“, a simple graphical tool for the evaluation of inter-laboratory comparisons*,

⁵⁾ D.Dopheide, B.Mickan, R.Kramer, H.-J.Hotze, J.-P.Vallet, M.R.Harris, Jiunn-Haur Shaw, Kyung-Am Park, *CIPM Key Comparisons for Compressed Air and Nitrogen, CCM.FF-5.b – Final Report*, 07/09/2006

⁶⁾ Cox M.G., *Evaluation of key comparison data*, Metrologia, 2002, **39**, 589-595

Nominal flow rate	SumChi(Obs)	Chiinv	ChiTest	KCRV		di	Ei (absolute)
				f_KCRV	u_KCRV		
[m ³ /h]				[m ³ /h]	[m ³ /h]	[m ³ /h]	
75	0.4442	3.8415	passed	74.5406	0.0210	0.0112	0.33
64	0.2677	3.8415	passed	63.9645	0.0180	-0.0074	0.26
32	0.6619	3.8415	passed	32.0395	0.0089	-0.0057	0.41
16	0.0045	3.8415	passed	15.9369	0.0044	-0.00023	0.03
8	0.0461	3.8415	passed	8.1042	0.0023	0.00039	0.11
1	0.7210	3.8415	passed	1.0180	0.0003	0.00020	0.42
0.5	1.5289	3.8415	passed	0.4926	0.0002	0.00012	0.62

6.2. Slovak Republic

Nominal flow rate	$Q_{V,20,dryAir}$	Uf	1/u ²	wi	chi	Pi	Data consistency
[m ³ /h]	[m ³ /h]	[m ³ /h]					
75	74.5231	0.0671	889.1864	0.3904	0.2708	0.99	OK
64	63.9761	0.0576	1206.5325	0.3901	0.1633	0.99	OK
32	32.0488	0.0288	4807.8677	0.3834	0.4081	0.98	OK
16	15.9372	0.0143	19442.3739	0.3836	0.0027	1.00	OK
8	8.1036	0.0073	75200.9398	0.3903	0.0281	1.00	OK
1	1.0177	0.0009	4768178.3692	0.4036	0.4300	0.97	OK
0.5	0.4922	0.0006	9768311.1009	0.2601	1.1313	0.83	OK

Nominal flow rate	SumChi(Obs)	Chiinv	ChiTest	KCRV		di	Ei (absolute)
				f_KCRV	u_KCRV		
[m ³ /h]				[m ³ /h]	[m ³ /h]	[m ³ /h]	
75	0.4442	3.8415	passed	74.5406	0.0210	-0.0174	0.33
64	0.2677	3.8415	passed	63.9645	0.0180	0.0116	0.26
32	0.6619	3.8415	passed	32.0395	0.0089	0.0092	0.41
16	0.0045	3.8415	passed	15.9369	0.0044	0.00038	0.03
8	0.0461	3.8415	passed	8.1042	0.0023	-0.00061	0.11
1	0.7210	3.8415	passed	1.0180	0.0003	-0.00030	0.42
0.5	1.5289	3.8415	passed	0.4926	0.0002	-0.00034	0.62

7. Degree of equivalence between laboratories

The 14th CCM meeting (February, 2013) recommended that pair-wise degrees of equivalence no longer to be published in the KCDB and that information on pair-wise degrees of equivalence published in KC reports be limited to the equations needed to calculate them, with the addition of any information on correlations that may be necessary to estimate them more accurately.

8. Summary and conclusion

The summary of inter-comparison results is mentioned down in the table:

Sonic nozzle		Laboratory	
Serial number	Nominal flow rate (m ³ /h)	Czech Republic CMI	Slovak Republic SMU
01508	75	passed	passed
JT-01-64000-1999	64	passed	passed
JT-01-32000-1999	32	passed	passed
JT-01-16000-1999	16	passed	passed
JT-03-8000-1999	8	passed	passed
JT-02-1000-1998	1	passed	passed
JT-02-500-1999	0.5	passed	passed
Mean		passed	passed