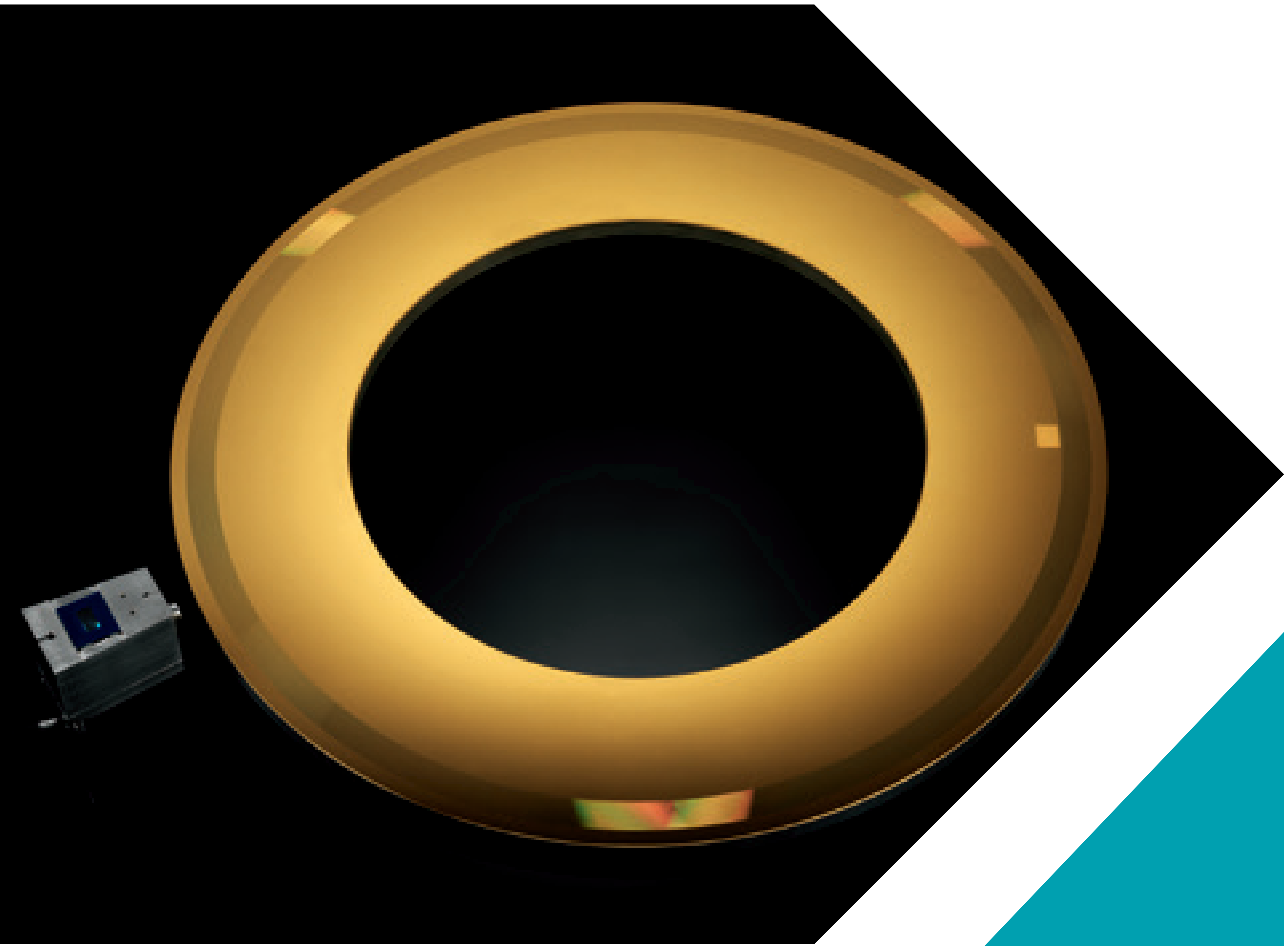
# Guidelines on the Calibration of Angular Encoders

EURAMET Calibration Guide No. 23
Version 1.0 (02/2018)



Length

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**EURAMET Calibration Guide No. 23**

Version 0.1 (02/2018)

# Calibration of Angular Encoders

**Purpose**

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibration of angular encoders.

## Content

# 1 INTRODUCTION

The aim of this document is to provide guidelines and improve harmonisation in calibration of angular encoders. It gives advice to calibration laboratories to establish practical procedures. The guideline is based on the knowledge produced under SIB58 Angles EMRP project [1], published papers (see list of references in section 9) and the VDI/VDE 2648 Part 1 (2009) written standard for calibration of angular encoders [2]. In the first part (sections 2, 3 and 4), the general definitions and the technical requirements for the calibration of angular encoders are given. The second part of this guideline is of procedural nature and gives practical advice to calibration laboratories. In sections 5 to 7 an example of a typical calibration procedure is presented. It is noted that laboratories working according to ISO/IEC 17025 shall validate their calibration procedures. This may lead to modification of the principles and examples given in this document.

# 2 SCOPE AND FIELD OF APPLICATION

This guideline refers to angle measuring instruments – angular encoders used to inform accurately on rotating angles in a wide range of applications.

Angular encoders are devices used to convert the angular position of a rotating shaft into an electrical signal. They are used in a wide range of applications where angular measurements to the accuracy level of the order of arcseconds are required, for instance in rotary tables on machine tools, high-performance robotics, servomotors, medical equipment, plotters, surveying instruments or accurate metrology.

The term angle encoder is typically used to describe encoders that have an accuracy of better than ± 5” and a line count above 10 000. Therefore, the precautions recommended in the guide for the calibration set-ups consider this accuracy level and provide distinction information for investigation of the angle encoder performance.

Although the encoders theoretically may be calibrated in the following positions:

- on-axis
- off-axis,

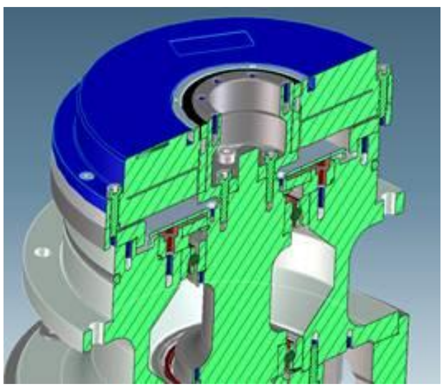
typical calibration of individual encoders is made ONLY in off-axis position, by the reasons explained below.

The on-axis calibration is used for angle encoders integrated into a rotary unit, for example into rotary tables (see Figure 2.1 (a)). So, in fact this is the calibration of a measuring machine and not of an individual encoder. The encoder is calibrated while mounted on the rotary unit and this defines actual performance of the angle encoder during operation (together with the mounted unit, in this case the rotary table). The calibration is carried out in the laboratory or mostly in the customer place (e.g. on-site).

The off-axis calibration is applied to individual angle encoders to define their general performance and characteristics. The encoder itself is delivered to the laboratory for calibration without being integrated into any other unit (see Figure 2.1 (b)). They may be accompanied with a display/counter unit which may also include interpolator in its electronics.

The off-axis calibration is mostly suitable for angle encoders with integral bearings. The system accuracy given by the manufacturers for these angle encoders includes the error of the integrated shaft coupling to the bearings. The examples for angle encoders with integral bearing are illustrated in Figure 2.1 for on-axis use in (a) and off-axis use in (b).

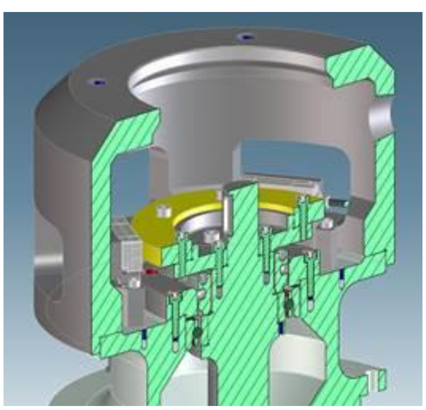
The angle encoders without integral bearing are designed for integration in machine elements or components and mostly consist of two components—a scanning head and a graduation carrier, which must be aligned to each other during mounting. The eccentricity of the shaft, as well as installation and adjustment, has a decisive effect on the achievable accuracy. Therefore, their accuracy heavily depends on the precision of the machine elements / components in which they are integrated as well as installation. Figure 2.2 illustrates angle encoders without integral bearing at on-axis and off-axis use. For these angle encoders, the laboratory has to integrate the item into their calibration system in order to perform off-axis calibration using the stricter specifications of the manufacturer.



(a)

(b)

**Figure 2.1**. Angle encoders with integral bearing for (a) on-axis use and (b) off-axis use.
*Courtesy of Fagor Automation.*



(a)

(b)

**Figure 2.2**. Angle encoders without integral bearing for (a) on-axis use and (b) off-axis use.
*Courtesy of Fagor Automation.*

For angle encoders without integral bearing, additional deviations resulting from mounting error in the bearing of the measured shaft, and adjustment of the scanning head, are expected. These deviations are not reflected in the system accuracy by the manufacturers. It is recommended here to calibrate such type of angle encoders as on-axis position in order to evaluate their performance better.

For angle encoders with integral bearing and integrated stator coupling (i.e. integrated shaft coupling to the bearings), manufacturer specifications usually also include the error due to the "integrated" shaft coupling. For angle encoders with integral bearing and "separate" shaft coupling, the angle error of the coupling must be added to the system accuracy of the encoder.

This guideline is intended for off-axis precise calibration of angular encoders with resolution down to milliarcsecond level, by comparison with another encoder normally installed within a turntable specifically designed for generating highly accurate angles. However, the guide also gives some short explanations and descriptions for use of other reference angle devices such as polygon-autocollimator and indexing tables. The users may utilise this information to create their own procedure.

## 2.1 Type of angular encoders

There are two basic types of angular encoders: Incremental and Absolute.

Incremental Encoders, also known as quadrature encoders or relative rotary encoders, are the simplest of the two position sensors.

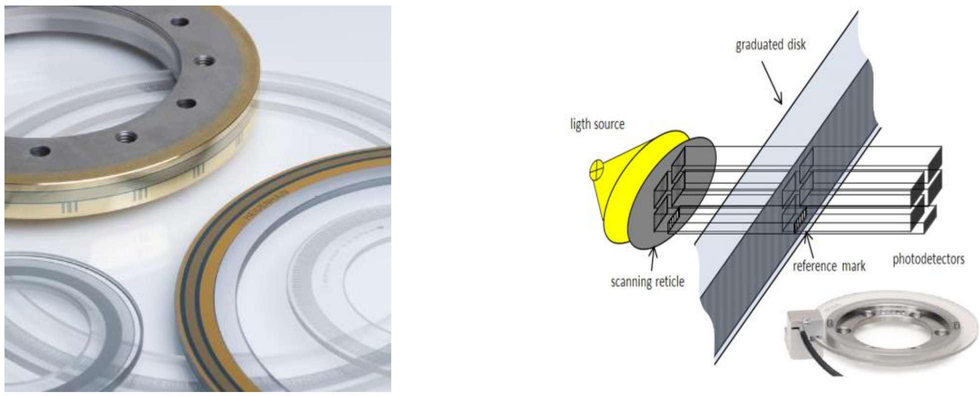
Absolute Position Encoders are more complex than quadrature encoders. They provide a unique output code for every single position of rotation indicating both position and direction. Their coded disk consists of multiple concentric "tracks" of light and dark segments. Each track is independent with its own photo detector to simultaneously read a unique coded position value for each angle of movement. The number of tracks on the disk corresponds to the binary "bit"-resolution of the encoder so a 12-bit absolute encoder would have 12 tracks and the same coded value only appears once per revolution.

**Figure 2.1.1**: Coded disc of an absolute encoder. *Courtesy of solo-labs.com. (https:// www.solo-labs.com/rotary encoders understanding practical implementation/)*

More recent absolute encoders make use of pseudorandom sequences to uniquely identify a position of the optical head with respect to the encoded circle.

Generally, absolute encoders have lower resolution compared with incremental encoders. This limit can be overcome by combining a pseudorandom code with a finer incremental structure, thus combining absolute positioning with high resolution. These encoders are not the subject of this Calibration Guide.

Most popular angle encoders are of optical type and work on the photoelectric scanning of a structured graduated disk. The graduations are applied to a carrier substrate of glass or steel by means of various photolithographic manufacturing processes. Some examples for graduated disks are illustrated in Figure 2.1.2 (a). When the glass disk is used, the light transmission is exploited, as depicted in Figure 2.1.2 (b).




(a) (b)

**Figure 2.1.2:** Angular encoders (a) absolute and incremental circular scales and scale drums (b) photoelectric measurement principle and circular scale with read-head at the right bottom corner. *Courtesy of Heidenhain.*

In that case, a beam of parallel rays of light flows through a scanning reticle and through the graduated disk itself. The scanning reticle has 4 scanning fields whose gratings are offset to one another by one fourth of a grating period each, plus one scanning field to be used as a reference mark. When the two graduations slide one above the other, periodic fluctuations of light intensity are observed. On the opposite side of the disk, the photo-detectors which are displaced in coincidence with the 5 scanning fields convert the light fluctuations in sinusoidal signals with a DC component, as shown in Figure 2.1.3.

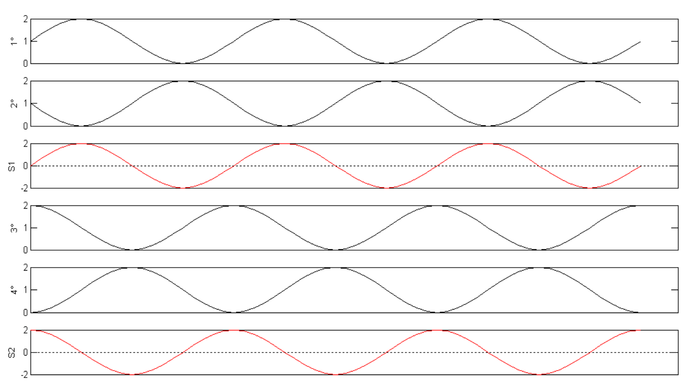


**Figure 2.1.3:** Sinusoidal signals with a DC component generated by the photodetectors.

Since the first and the second fields of the scanning reticle have a phase difference of half a period, the signal generated by the second photodetector is phase shifted by 180°

with respect to the signal generated by the first one. Hence, the DC component can be eliminated by evaluating the difference between the two signals, resulting in a sinusoidal signal S1 symmetrical around the zero line. Similarly, the third and the fourth fields generate a sinusoidal signal S2 which is phase shifted by 90° with respect to S1.

So finally, $S_1=A_0 \sin \varphi$ and $S_2=A_0 \cos \varphi$, being $\varphi = 2\pi x/T$, where $T$ is the signal period and $x$ is the displacement.

On the contrary, when a steel disk is used instead of a glass disk, a method based on the reflected light is exploited. In this case, the graduations are a series of reflecting and absorbing lines and the photo-detector is on the same side as the light source.

The number of transparent and dark segments or slots on the disk determines the resolution of the device and increasing the number of lines in the pattern increases the resolution per degree of rotation. Typical high performance graduated discs have a resolution of up to tens of thousands of pulses per rotation.

The angular measurement accuracy of a rotary encoder is highly dependent on the grating pattern manufacturing error (uniformity and eccentricity of the graduation), glass disk installation on the measured rotary axis, the scanning head(s) alignment and signal processing circuits. All these elements contribute to rotary encoder errors. Therefore, in order to achieve the highest accuracy, such repeatable errors need to be removed or corrected through software mapping after the calibration process.

### 2.2 Laboratory environment / Site of calibration

The encoder shall be calibrated in a laboratory that provides adequate control for environmental conditions (stable ambient temperature, low vibration, and a stable laminar air flow). In order to achieve the required uncertainty for high resolution angular encoders (e.g. less than 0.1”), experience shows that the temperature stability within ±0.3 ºC (in the laboratory) is needed.

### 2.3 Calibration conditions

During the calibration the test encoder must be operated according to the manufacturer’s instructions. When calibrating an encoder by comparison against a reference one, both encoders must be connected by a rigid and backlash–free coupling shaft, paying attention to the concentric and coaxial position of the axes of rotation and the backlash-free fixing of the housing. If it’s possible the coupling devices of the test encoder should be used during its calibration.

The calibration should always be carried out by maintaining the eccentricity and axial parameters within the specifications of the manufacturer and, if possible, close to the conditions that will apply later when used.

## 3 TERMINOLOGY

### 3.1 Encoder’s Eccentricity Error

Eccentricity error is the radial distance between the grating’s axis of rotation and the centre of the rotary grating, see Figure 3.1

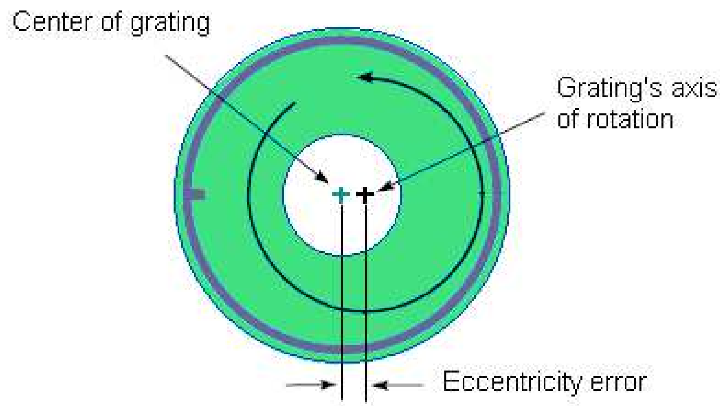



**Figure 3.1** Internal eccentricity error of an encoder
*(Courtesy from MicroE Systems, Manual TN-Alignment_of_Rotary_Scales)*

Eccentricity is the largest controllable source of error in any rotary encoder system. To achieve the highest level of long range accuracy, the eccentricity must be minimized. The run-out of the bearing shaft can degrade the long range rotary accuracy, making the selection of the proper bearing an important part of the integration process. The effect of eccentricity and run-out has an increasing effect as the grating size decreases.

## 3.2 Rotary Accuracy

Rotary accuracy can be divided into two different parts; long range and short range accuracy. The long range accuracy is typically measured over 360° and the short range accuracy is measured over a smaller angle range. The major error sources for long range accuracy are: eccentricity error and bearing run out. The major error sources for short range accuracy are: small imperfections in grating pattern, variations in the sensing components, and imperfection in signal corrections.

## 3.3 Notation

The following notation will be used in further calculations:

$\underline{\alpha_{\mathrm{REF}}}$ reference angles generated by the standard turntable, taken as conventional true values. $\underline{\alpha_{\mathrm{REF}}}$ is the difference between the readouts in both final and initial measurement positions. For full circle calibrations normally $\alpha_{\mathrm{REF}_1} = 0$

$$\alpha_{\mathrm{REF}} = \alpha_{\mathrm{REF}_2} - \alpha_{\mathrm{REF}_1}$$

$\underline{\alpha_{\mathrm{CAL}}}$ angles indicated by the encoder under calibration. $\underline{\alpha_{\mathrm{CAL}}}$ is the difference between the readouts in both final and initial measurement positions. For full circle calibrations normally $\alpha_{\mathrm{CAL}_1} = 0$

$$\alpha_{\mathrm{CAL}} = \alpha_{\mathrm{CAL}_2} - \alpha_{\mathrm{CAL}_1}$$

These angles are affected by some deviations with respect to the nominal values due to internal mounting errors (eccentricity and run-out of the grating with respect to the encoder's body) and external mounting errors with respect to the standard turntable to which the encoder is coupled for the calibration process (axis-axis eccentricity and tilt angle between both axes).

$\Delta\alpha$ indication error (measurement error, according to VIM). $\Delta\alpha = \alpha_{CAL} - \alpha_{REF}$. Difference between the angle indicated by the test encoder and the one indicated by the reference encoder, after rotating an assumed angle.

$\delta_i$ relevant components influencing the measurement

$N$ number of relative positions between both encoders;

$n$ index referred to the relative position between encoders;

$L$ number of measurement series in each relative position;

$l$ index referred to the measurement series in each position;

$M$ number of readings in each measurement point;

$m$ index referred to the reading in each measurement point;

$J$ number of calibration points on each encoder;

$j$ index referred to the calibration point.

# 4 EQUIPMENT AND DEVICES

Comparison methods are the most widely used calibration methods and rely on a more accurate measurement reference to calibrate a less accurate encoder. So, comparison-calibration methods are limited to their reference angle's accuracy. Some angle references only cover a small portion of a single rotation, so the calibration needs to be performed in multi steps. As a result, the process will be time consuming and less accurate.

Because these are off-axis calibration methods, recalibration of the encoder in its final installation is not possible. As a result, the comparison calibration methods cannot capture the variations due to changes in mounting and operating conditions, such as speed and temperature.

## 4.1 Reference standards / devices (systems)

Reference standards / devices traceable to SI unit radian [16] shall be used.

For encoder calibration, the main used reference standards are turntables with a measurement range covering 360º and capability of positioning in any angle as a multiple of a small specific step, dependent on each turntable, which limits its resolution.

They are typically built around internal high accuracy angular encoders, with more than 36 000 steps, electronic interpolation and, in some cases, several optical reading heads, making possible to get angular resolutions down to level of milliarcseconds.

The method for achieving traceability to SI unit radian is based on a subdivision of the natural and error-free standard of the full circle, 360° = 2π rad. These tables must be calibrated and fully investigated prior to use for calibration of encoders. The method for investigations and calibration of the angle reference systems vary with their types.

### 4.2 Auxiliary devices

Auxiliary devices are used to facilitate the encoder calibration process. Some auxiliary devices with different properties affect the calibration results. Therefore, they need to be reported in the calibration certificate.

When calibrating an encoder by comparison against a reference one, both encoders must be connected by a rigid and backlash–free coupling shaft, paying attention to the concentric and coaxial position of the axes of rotation and the backlash-free fixing of the housing. When possible the coupling device of the test encoder should be used during its calibration. Figure 4.2.1 illustrates some of the coupling elements.

The calibration should always be carried out maintaining the eccentricity and axial parameters within the specifications of the manufacturer.

Encoders must be calibrated with the delivered display units, counters and also the interpolators used in the electronics. On the other circumstances, the customer must be informed and any other devices (e.g. interpolators of the laboratory) used in the calibration have to be specified with the selected resolution (i.e. interpolation rate) in the calibration certificate since they influence the calibration results significantly.

(a) (b)

**Figure 4.2.1.** Examples for shaft couplings (a) Diaphragm coupling (b) Flat coupling.
*Courtesy of Fagor Automation*.

## 5 CALIBRATION PROCEDURES

For calibration of encoders in large range the use of cross-calibration methods is suitable [17]. In that method, a second artefact (e.g. optical polygon with autocollimator, indexing table/rotary table or another angle encoder) is used and a relative error is measured as the difference between the indications of both artefacts. This method permits calibrating both angle standards together. Although this method provides calibration of both devices (test and reference) simultaneously, it is recommended to investigate the reference in advance e.g. in terms of interpolation errors, its repeatability/stability etc. in order to provide better knowledge of the test encoder and uncertainty of calibration.

Other possibility is using a direct comparison method against well known calibrated reference standards such as optical polygons, indexing tables / rotary tables or angle encoders and performing corrections using the certified deviation values of the reference system. Reference angle encoders may be utilised while they are fitted to a turntable (so called rotary tables) or as stand-alone but mounted on a suitable fixture. Both cases need detailed investigations prior to use.

Calibration of an encoder by using a second encoder taken as a reference has more advantages than using a polygon due to the similarity of instruments and the higher resolution offered by the angle encoder with respect to the polygon. While a 36 sided polygon offers 10 degree steps, any encoder can offer steps with values of a few arcseconds and even tenths of arcsecond, so helping to know in smaller ranges the deviations of the encoder under calibration.

## 5.1 Calibration with limited resolution steps (using a polygon / indexing table)

The encoder may be calibrated using an optical polygon. Cross calibration or direct comparison can be applied. It is necessary to use an autocollimator for the readings and take care of the pyramidal errors of the polygon in the alignment process. Of course, the number of steps in the calibration is limited because the lower number of positions reachable with the polygon, just the number of its faces, in comparison with using a second encoder. So, with 24 or 36 sided polygons, minimum steps of 15 or 10 degrees are achievable respectively. A schematic set-up utilising a polygon-autocollimator is illustrated in Figure 5.1.1 as an example.

The problems associated with using a polygon are the lack of flatness and pyramidal errors of their faces, together with alignment and errors coming from the autocollimator used for the calibration, which are not present in the case of using a second encoder as standard. As a consequence, the uncertainty reached is higher than in the case of using a reference encoder.

The test encoder can also be calibrated using an indexing table, schematic set-up of which is given in Figure 5.1.2. The calibration steps are limited by the indexing resolution. Since the indexing tables operate with a lifting mechanism, the user must consider this and use a special coupling between indexing table and angle encoder coping with this limitation. Special coupling elements allowing e.g. 1 mm axial movement (also providing rigid and backlash–free rotation movements) are provided by the manufacturers.

For investigation of the interpolation errors of the angle encoder, the set-up shown in Figure 5.1.1 may be utilised and the interpolation errors are measured using the autocollimator by measuring the much smaller steps than the polygon nominal angles. A similar approach may also be used for the set-up shown in Figure 5.1.2 by simply adding a plane mirror and autocollimator to the set-up with a special tilt/rotation mechanism (which already exists in some indexing tables).

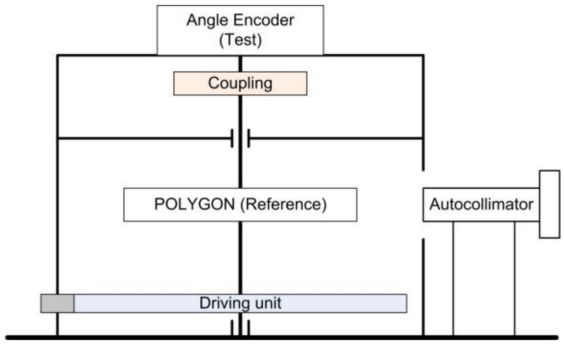



**Figure 5.1.1.** A schematic set-up utilising polygon-autocollimator for calibration of test angle encoders at off-axis.

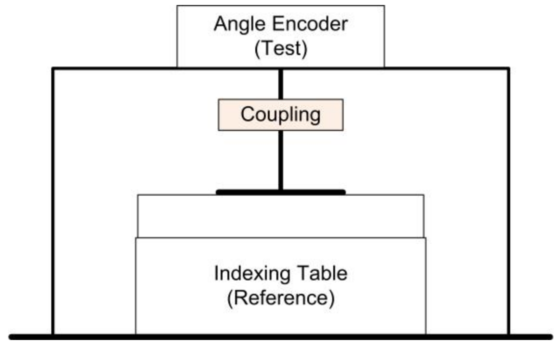



**Figure 5.1.2.** A schematic set-up utilising an indexing table for calibration of test angle encoders at off-axis.

The polygons are very robust standards and utilised widely for calibration of angle encoders when fitted to rotary tables (i.e. on-axis calibration). They are also perfectly suitable for on-site calibrations utilising the autocollimators to take readings either in null detection or absolute readings. In this case, the schematic arrangements shown in Figure 5.1.1 may be used as up-side down view.

## 5.2 Calibration with high resolution steps (using an angle encoder / rotary table fitted with an angle encoder)

The test encoder is calibrated using a reference angle encoder or a rotary table fitted with an angle encoder (see figures 5.2.1 and 5.2.2 for schematic views). Direct or cross calibration may be utilised for both cases.

To minimize the adjustment effects between the reference encoder measurement system and the test encoder, the latter is fixed at different relative angular positions with respect to the reference encoder. This strategy permits also separate long periodic effects caused by

(a) systematic errors of encoder (with only one reading head) and/or

(b) “errors” of calibration (adjustments outside of specification)

At each relative position the relative error is measured along a full rotation and subsequent calculation of the mean is done. It must be noted that this has to be done for cross calibration in the number of cross calibration steps.

The biggest advantage of using angle encoder or RT fitted with angle encoder is to achieve calibrations with a higher number of calibration steps up to the number of graduation lines around the encoder or even up to larger values such as the encoder resolution. It is also possible to take readings in dynamic mode (while both encoders are rotating).

During the calibration the test encoder must be operated according to the manufacturer’s instructions and, if possible, under the same or similar conditions on which the encoder is going to work later.

The calibration should always be carried out by maintaining the eccentricity and axial parameters within the specifications of the manufacturer.

This is the method of calibration described in the present procedure.

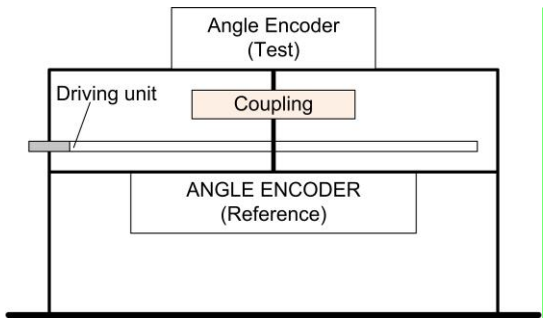



**Figure 5.2.1.** A schematic set-up utilising a reference angle encoder for calibration of test angle encoders at off-axis.

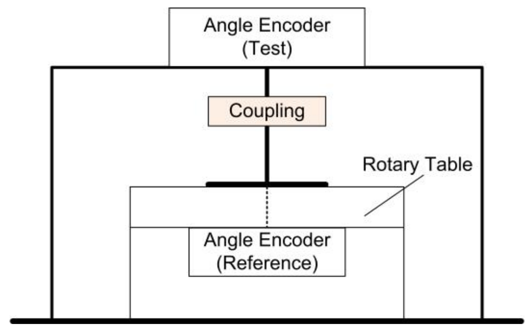



**Figure 5.2.2.** A schematic set-up utilising an indexing table for calibration of test angle encoders at off-axis.

## 5.3 Handling, preparation and adjustments

Before proceeding with the calibration, the following steps shall be taken:

- Familiarize yourself with the functioning, handling and features of the test encoder by consulting its manual.
- Familiarize yourself with the mounting manufacturer's tolerance specifications.
- Connect the test encoder with the reference encoder by a rigid and backlash-free coupling shaft. It's important to pay attention to the concentric and coaxial position of the axes of rotation and the backlash-free fixing of the housing, for that it's useful to use a device to align the test encoder connected with the reference encoder and to change the relative position between the test and the reference encoder.
- Check the operability of the encoder.
- Allow approx. 24 hours for the thermal adaptation of the encoder to your laboratory environment.
- Start-up the reference and the test encoder at least 2 hours before the beginning of the measurements to enable an adequate warming-up.
- Align the test encoder with the reference encoder for getting axial run-out body, axial run-out axis, axial run-out body/axis, radial run-out body and radial run-out axis within the manufacturer's tolerance specifications or under the working conditions specified by the test encoder user. For that it's possible to use inductive probes for checking the alignment around the full range.

**Figure 5.3.1** Example of a device at CEM to align the test encoder with the reference encoder

- Select the number of calibration points. This number shall be agreed with the customer.
- Select the number of relative positions between both encoders. At least three positions are recommended (see Note 1).
- Select the number of measurement series in each relative position and the number of readings in each measurement point.
- In the first relative position fix the zero for the test and the reference encoders. After the series of the first relative position, change the relative position between both encoders and make the new measurement series for the new relative position.

*Note 1:*
It is recommended to calibrate the angle encoders in different relative angular orientations with respect to the primary standard to minimize the influence of residual systematic errors of the primary standard. It is even more important to detect and characterise errors due to the mechanical coupling of the devices. Usually, the primary angle standards have been characterised quite well. Errors due to the mechanical coupling of the systems can be differentiated from residual errors of the primary standard by their change in phase when the relative angular orientations between the primary standard and the test encoder are changed.

Therefore, it is recommended not to change the adjustment between measurements in different relative positions in order to observe the actual adjustment (i.e. influence of the adjustment) during the calibration. This is interpreted as follows: *If a first harmonic is seen in the encoder deviation, the reason could be a systematic error of the encoder or the influence of the calibration set-up. For separation, if a shift is observed between the results in the different relative positions, the reason is the set-up; if a shift of the long periodic part is not seen, the reason is the encoder itself (systematic error by using only one reading head).*

### 5.4 Determination of the encoder deviations and calculation of data sets

In general, the result for each point of calibration *j*: is the deviation $\Delta\alpha$ of the angle measured by the test encoder from the angle provided by the reference system according to the following mathematical model

$$\Delta\alpha = \alpha_{\mathrm{CAL}} - \alpha_{\mathrm{REF}} + \sum_{i=1}^{I} \delta_i \tag{1}$$

with

$\Delta\alpha$: the angle deviation of the encoder under test,

$\alpha_{\mathrm{CAL}}$: the angle measured by the encoder under test, and

$\alpha_{\mathrm{REF}}$: the angle measured by the reference encoder.

$\delta_i$: relevant components influencing the measurement

*Note 2:*
For the case of incremental encoders without or with more than one reference mark ZERO, there are different start positions after switching on the encoder as the measurement starts with one of these reference impulses. This means that there are no reproducible "nominal" values in the calibration and later in the application. In this case, issuing a certificate with results of discrete measurement positions and corresponding deviations is not possible and also the customer cannot use the certificate for applying corrections. In such cases, the result could be expressed, for example, as a range: e.g. *Deviations of the angle encoder are within +/- 30" in the full range, 360°*. This gives overall information about the angle encoder performance.

For the final calibration value, $\Delta\alpha$, multiple measurements may be obtained and processed by repeating measurements in several relative positions between both encoders.

The full calibration requires $n_r = NLMJ$ independent individual measurements performed at *N* relative angular positions between both encoders, with *L* repetitions (in back and forth senses) per position, *J* steps per rotation sense (for sampling the behaviour of the encoder under test) and *M* readings per point.

- Note 5.4.1: Number *M* of data taken at each setting may vary depending on the used software and also the encoders' stability and resolution. In case of dynamic measurements *M* = 1.
- Note 5.4.2: The aim for performing calibrations at different relative angular positions between both encoders is to minimize the residual error sources, mainly due to eccentricity and tilt angles between rotation axes.

*N* Number of relative positions between both encoders. Such number depends on the uncertainty objective; for example, PTB calibration of a RON 905, with *u*=0,01" is realised in 12 relative positions (30°). But at least a minimum of 3 relative positions is recommended. To minimize the influence of the harmonics it is advisable to choose 3 coprime shifted positions, for example 0-0, 0-55, 0-240.

*L* number of measurement series in each relative position. At least 6 series (3 back and 3 forth) for each relative position are advisable.

*J* number of calibration points on each encoder shall be preferably arranged with the customer. With a higher number of points, there will be a better knowledge on the small range behaviour the encoder but the calibration time and its cost will be also higher. It is advisable to choose 12 points corresponding to 30 degrees in the full range.

*M* number of readings in each measurement point. This depends on the stability and noise of the electronics. Depending on this, the value of 10 to 100 may be chosen. But at least 10 readings are advisable in each point. Again, in case of dynamic measurements $M = 1$.

### 5.5 Determination of encoder deviations in small range. Investigation on interpolation errors.

If the points of calibration of one encoder are chosen as multiples of its basic resolution it will be necessary to evaluate the interpolation error to know the behaviour in steps out of such basic resolution.

An important prerequisite for the calibration accuracy is the high resolution which can be achieved by applying an interpolation procedure to the basic signal period of the scanning signal (e.g. 7.2” or 36” etc). With an interpolation factor of 8192, a resolution of ~0.0009” is achieved for a 7.2” basic resolution. However, one must take into account the accuracy of the interpolation procedure applied to the signal period of the scanning signal. This is because position errors within one signal period of 7.2” become apparent in very small angular motions and in repeated measurements. These errors within one signal period are caused by the quality of the sinusoidal scanning signals and their subdivision. The factors influencing the result are summarised by the encoder manufacturers as follows [15]:

- the size of the signal period
- the homogeneity and edge definition of the graduation
- the quality of the optical filter structures
- the characteristics of the photoelectric detectors

the stability and dynamics during the further processing of the analogue signals

The manufacturers usually state that their angle encoders take these factors of influence into account and permit interpolation of the sinusoidal output signal with subdivision accuracies of around ± (1-2) % of the signal period. This is about 0.1” (0.015 × 7.2”) using the manufacturer specifications.

In addition to comparison with the reference angle encoder or RT’s angle encoder readings, different methods could be used for precise determination of encoder deviations in small range, particularly so called interpolation errors.

- The laboratories may use a precise high resolution electronic autocollimator. The angle encoder readings can be compared with the autocollimator readings just like in autocollimator calibration through a reference mirror located on the rotary table or direct from the polygon face for section 5.1.
- The laboratories may use a reference angle interferometer. The angle encoder readings can be compared with the interferometer readings just using the angular

retroreflectors located on the suitable place connected to test encoder or on the rotary table for on-axis calibration.

- In order to achieve smaller uncertainty, error-separating shearing techniques shall be also applied. This technique offers a unique opportunity to separate errors of devices without recourse to any external standard. They were first adapted by PTB to the calibration of autocollimators with angle encoders (i.e. Angle Rotary Tables). The achieved standard uncertainty is about 1 milliarcsecond (5 nrad) [3, 4]. The results showed that the shearing method is ideally suited for the calibration of interpolation errors of the devices at small angular scales which are difficult to characterise with other methods.
- The laboratories may also use precise displacement sensors such as capacitive sensors or inductive probes which do not show interpolation errors unlike displacement probes utilising linear encoders. By placing the displacement sensors on the circumference of the angle encoder or even in the larger circumference, displacement values may be converted to angular values using radius of this circumference and compare with the test encoder reading values.

# 6 EVALUATION OF THE RESULTS

Calibration results may be used for conformity assessment of the manufacturer's specifications but for the customer specific use as a traceable link to the SI (on demanded conditions) the results shall be evaluated by the users according to their needs.

# 7 MEASUREMENT UNCERTAINTY

The standard measurement uncertainty should be evaluated according to the Guide to the Expression of Uncertainty in Measurement [18]. Alternatively, the laboratories may choose to use the approach according to the Supplement 1 to the GUM [19] by propagating distributions (to obtain the Probability Density Function – PDF – of the output quantity from which an estimate of the output quantity itself, the standard uncertainty associated with it, and the coverage interval for a given coverage probability can be derived). In this section, the standard approach is outlined.

## 7.1 Mathematical model

The results of the calibrations are the differences between the indications of the test encoder and the reference one, after rotating by assumed angle. In the mathematical measurement model the sum of various corrections are also taken into account.

The mathematical model for each point of calibration, $j$ is as follows:

$$\Delta\alpha = \alpha_{\mathrm{CAL}} - \alpha_{\mathrm{REF}} + \sum_{i=1}^{I} \delta_i \tag{1}$$

where

$\Delta\alpha$ indication (measurement) error;

$\alpha_{\mathrm{CAL}}$ the value of the angle indicated by the encoder calibrated, which is the difference between the readouts obtained in both measurement positions $\alpha_{CAL2}$ and $\alpha_{CAL1}$;

$$\alpha_{CAL} = \alpha_{CAL_2} - \alpha_{CAL_1} \quad (2)$$

$\alpha_{REF}$ the value of the angle indicated by the reference encoder, which is the difference between the readouts obtained in both measurement positions $\alpha_{REF2}$ and $\alpha_{REF1}$;

$$\alpha_{REF} = \alpha_{REF_2} - \alpha_{REF_1} \quad (3)$$

$\delta_i$ relevant components influencing the measurement.

### 7.2 Equation of the measurement uncertainty

According to the mathematical model given in (1), the equation of the measurement uncertainty is as follows:

$$u_c^2(\Delta\alpha) = c_1^2 u_1^2(\alpha_{CAL}) + c_2^2 u_2^2(\alpha_{REF}) + \sum_{i=1}^{I} c_i^2 u_i^2(\delta_i) \quad (4)$$

where $c_1$, $c_2$ and $c_i$ are relevant sensitivity coefficients.

### 7.3 Description of the input quantities

In the Guide [18] there is a statement that says: "The uncertainty in the result of a measurement generally consists of several components which may be grouped into two categories according to the way in which their numerical values are estimated: A. those which are evaluated by statistical methods, B. those which are evaluated by other means".

#### 7.3.1 Type A

Taking into account that the final results of the calibration are the mean values of different readings, series and relative positions, the best evaluation of the type A component is obtained after applying an analysis of the variance (ANOVA) to verify if the values obtained through the different relative positions have interseries effects. If interseries variability is significantly higher than the intraseries one, it is very likely that equation (8) below underestimates significantly the uncertainty of the mean value $\Delta\alpha$ (7).

In Paragraph 3, the following notation was anticipated for further calculations:

- $N$ — number of relative positions between both encoders;
- $n$ — index referred to the relative position between encoders;
- $L$ — number of measurement series in each relative position;
- $l$ — index referred to the measurement series in each position;
- $M$ — number of readings in each measurement point;
- $m$ — index referred to the reading in each measurement point;
- $J$ — number of calibration points on each encoder;
- $j$ — index referred to the calibration point.

It is considered that measurements taken in the same point within a common relative position **do not have interseries effect.** The possibility of existence of such effects will be analyzed only for the case of measurements taken in different relative positions between encoders.

Calling $\Delta\alpha$ the value of the angular error, for the relative position $n$, the mean value is defined as

$$\Delta\alpha_n = \frac{1}{LM}\sum_{l=1}^{L}\sum_{m=1}^{M}\Delta\alpha_{lm} \tag{5}$$

It is considered that the $LM$ measurements belong to the same population.

$$s_n^{\,2}(\Delta\alpha_n) = \frac{1}{(LM-1)}\sum_{l=1}^{L}\sum_{m=1}^{M}\left(\Delta\alpha_{n_{lm}} - \Delta\alpha_n\right)^2 \tag{6}$$

The mean value and the experimental variance of all measurements, considering all relative positions will be:

$$\Delta\alpha = \frac{1}{N}\sum_{n=1}^{N}\Delta\alpha_n \tag{7}$$

$$s^2(\Delta\alpha) = \frac{1}{(N-1)}\sum_{n=1}^{N}\left(\Delta\alpha_n - \Delta\alpha\right)^2 \tag{8}$$

In ANOVA the first estimation denoted as $s_a^2$ is obtained from the equation:

$$s_a^2 = LMs^2(\Delta\alpha) = \frac{LM}{N-1}\sum_{n=1}^{N}\left(\Delta\alpha_n - \Delta\alpha\right)^2 \tag{9}$$

This estimation has $\nu_a = N-1$ degrees of freedom.

The second estimation, $s_b^2$, is simply the average of the $N$ intraseries variances:

$$s_b^2 = \overline{s_n^2} = \frac{1}{N}\sum_{n=1}^{N}s_n^2 = \frac{1}{N(LM-1)}\sum_{n=1}^{N}\sum_{l,m=1}^{L,M}\left(\Delta\alpha_{n_{lm}} - \Delta\alpha_n\right)^2 \tag{10}$$

This estimation has $\nu_b = N(LM-1)$ degrees of freedom.

The calculation of $s_a^2$ is based on the variation of the mean values, while $s_b^2$ is based on the variation of the observations within each relative position. So, the difference among them indicates the presence of some effect varying from series to series but staying constant within each series. To verify this possibility the *F-test* is used.

Distribution $F$, as in equation (11),

$$F(\nu_a, \nu_b) = \frac{s_a^2(\nu_a)}{s_b^2(\nu_b)} \tag{11}$$

is the probability distribution of the ratio between two independent estimations of the variance $\sigma^2$ of a random variable uniformly distributed. $F$ is tabulated for different significance levels and degrees of freedom. If, for $\nu_a$, $\nu_b$ and a chosen significance level, the $F$ result is bigger than the ratio $s_a^2(\nu_a)/s_b^2(\nu_b)$, the conclusion is that THERE IS NOT an interseries effect statistically significant. Otherwise it is inferred that such effect exists.

The methodology used to calculate the uncertainties in each case is as follows.

**a)** <u>Case on which **THERE IS NOT** interseries effect</u>

On this case, the variance obtained by Type A evaluation is

$$u_j^2 = \frac{Ns_a^2(\Delta\alpha) + N(LM-1)s_b^2}{NLM(NLM-1)} \tag{12}$$

**b)** <u>Case on which **THERE IS** interseries effect</u>

If the existence of an interseries effect is admitted, then it is useless to increase indiscriminately the number of repetitions within each relative position; so

$$u_j^2 = \frac{s^2(\Delta\alpha)}{N} \tag{13}$$

Once the uncertainty of each calibration point of the encoder has been calculated – $u_j^2$, it is possible to evaluate the type A contribution of the full uncertainty of the encoder as:

$$u_A^2 = \frac{1}{J}\sum_{j=1}^{J} u_j^2 + \frac{1}{J-1}\sum_{j=1}^{J}\left(u_j - \overline{u_j}\right)^2 \tag{14}$$

### 7.3.2 Type B

As type B components, various quantities are taken into account.

#### 7.3.2.1 Resolution of the test encoder and reference encoder

The component connected with the resolution of the encoder under calibration is calculated using the formula:

$$u_{\text{resCAL}} = \frac{\alpha_{\text{CAL}}}{2\sqrt{3}} \tag{15}$$

where $\alpha_{\text{CAL}}$ is the resolution of the encoder being calibrated.

The value of $u_{\text{resREF}}$ for the reference encoder is calculated using the same formula (15) with $\alpha_{\text{REF}}$ being the resolution of the reference encoder.

#### 7.3.2.2 Uncertainty of the reference encoder calibration

The value of this component is taken from the calibration certificate of the reference encoder.

$$u_{\mathrm{cerREF}} = U_{\mathrm{cerREF}}/k \tag{16}$$

where $k$ is the coverage factor given in the calibration certificate of the reference encoder.

#### 7.3.2.3 Interpolation error of the reference encoder

Each precise reference encoder should have the value of the interpolation error determined. This error, a systematic one, is connected with the process of interpolation of the signals obtained from the lines of encoder's circular disc. Since they are usually larger than the resolution of the reference angle encoder, their contribution is expected to be high particularly for high precision angle encoder calibration.

The interpolation error value can be determined using a very precise autocollimator [3], capacitive/inductive sensors [4] or more precisely using the shearing methods [5, 11 and 12]. Alternatively, this value can be determined using the manufacturer specifications of the reference angle encoder. The manufacturers usually state 1 % to 2 % of the basic resolution value as interpolation error (if no compensation is made) [13].

The value of this error (including its uncertainty if defined experimentally) should be put into the uncertainty budget, as follows:

$$u_{\mathrm{intREF}} = \frac{e_{\mathrm{intREF}}}{\sqrt{3}} \tag{17}$$

where $e_{\mathrm{intREF}}$ is the value of the interpolation error of the reference encoder.

#### 7.3.2.4 Influence of the residual eccentricity and the residual inclination angle between axes of rotation (run-out errors)

To minimize the influence of the eccentricity and the inclination angle between test and reference angle encoders, the tested encoder should be mounted and coupled to the reference one as precisely as possible according to the mounting tolerances given in the producer's mounting instructions, if available. In addition, as indicated in 2.1, calibrating in several relative positions of the two encoders (the reference one and the one under calibration) and taking the average, reduces such influence and could be omitted.

In this case, errors due to the influence of the residual eccentricity together with the influence of the residual inclination angle between axes of rotation (i.e. run-out errors) can be derived experimentally from the differences of the means of the relative positions and included in the uncertainty budget. These differences contain both the effect of the angle between axes of rotation and the effect of the eccentricity (between test and reference axis). In other words, the differences enter only once into the measurement uncertainty budget. The term below could be applied for this unique component.

$$u_{\mathrm{aba}} = \frac{e_{\mathrm{aba}}}{\sqrt{3}} \tag{18}$$

where $e_{aba}$ is the error value for influence of residual angle and eccentricity between axes of rotation (test and reference), determined 'experimentally'.

So, measured or estimated eccentricity value and inclination angle (i.e. non parallelism of the measurement planes between test and reference encoder because an axial run-out) are used to calculate the error contribution. The standard deviation, calculated from all measurements in different relative positions is a experimental value of this influence. Theoretical explanations for this effect are given in Appendix C.

### 7.4 Combined standard uncertainty

The combined standard uncertainty is now calculated as:

$$u_c = \sqrt{\left(c_1^2 u_A^2 + c_2^2 u_{resCAL}^2 + c_3^2 u_{resREF}^2 + c_4^2 u_{cerREF}^2 + c_5^2 u_{intREF}^2 + c_6^2 u_{aba}^2\right)} \quad (19)$$

### 7.5 Expanded uncertainty

The expanded uncertainty is calculated from the following formula:

$$U = k \times u_c \quad (20)$$

where $k$ is the value of the coverage factor for a coverage probability of 95 %.

It could be assumed that the probability distribution is the $t$-distribution with the $t$ factor based on the effective degrees of freedom $v_{eff}$. The value of the effective degrees of freedom is calculated from the Welch-Satterthwaite formula:

$$\upsilon_{eff} = \frac{u_c^4}{\sum_{i=1}^{N} \frac{u_i^4(y)}{\upsilon_i}} \quad (21)$$

The calculated value $v_{eff}$ is the base for finding the value of the coverage factor $k$ (from the table which can be found in Guide to the expression of Uncertainty in Measurement (GUM) [18]).

## 8 CALIBRATION CERTIFICATE

The certificate of calibration shall contain the following information:

- The manufacturer, type and serial number of the encoder;
- The settings during calibration (measuring range, resolution, number of steps and repetitions, type of coupling and its features, etc.);
- Display/counter unit used with its serial number. Interpolator specifications if calibration Lab. uses their own electronics;
- The ambient temperature range during the calibration;
- The measurements results may be presented in tabular form (see Appendix A). Remarks for the results; e.g. reduced measurement deviation (e.g. referred to mean value 0") in a specific small range shall be written;

- The uncertainty of the calibration, with indication of the coverage factor and/or the level of confidence.

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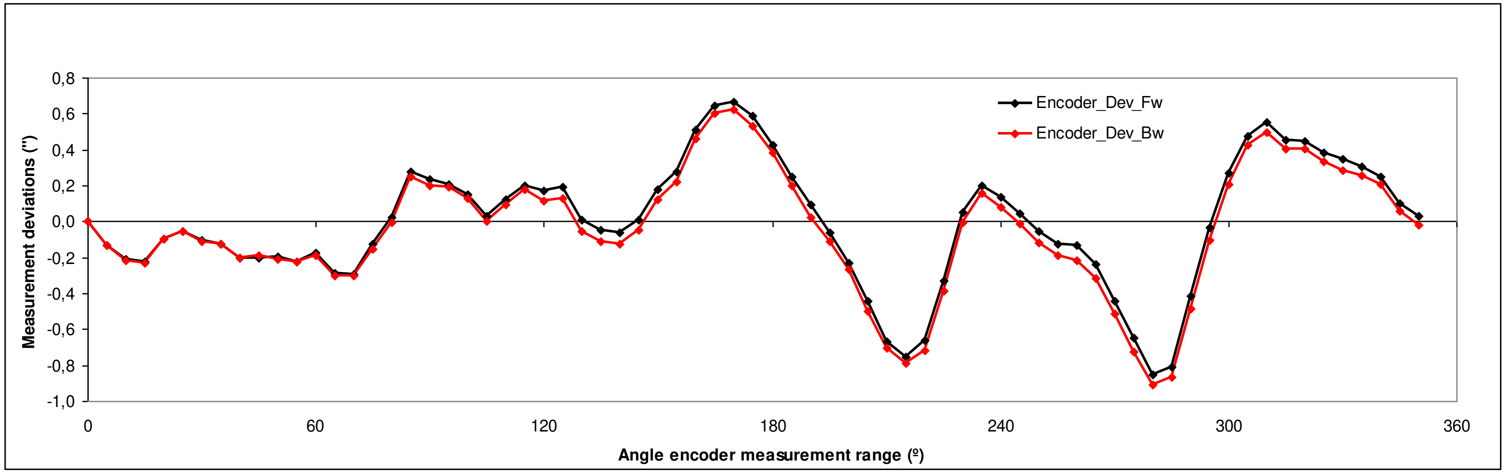
# 1 APPENDIX A: Example of DATA sets for calibration of encoders

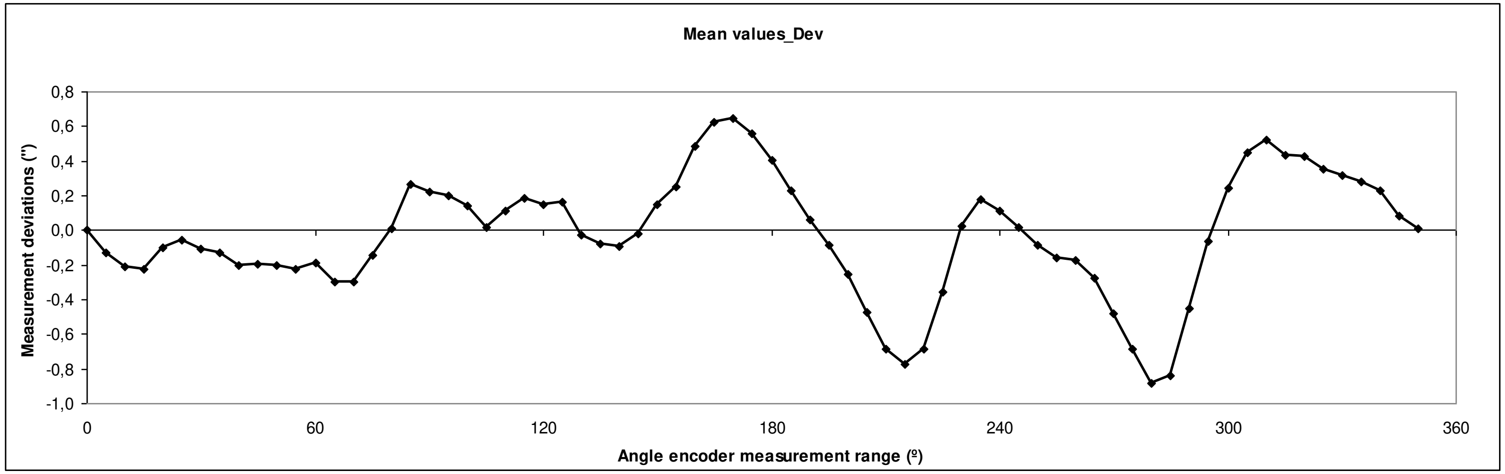
## *N* = 3 relative coprime positions (0-0, 0-55, 0-240)

| **Results taken at forward direction** **$N$ = 3, $L$ = 3 forward, $M$ = 5, $J$ = 72** | | | | | ***Results taken at backward direction*** ***$N$ = 3, $L$ = 3 backward, $M$ = 5, $J$ = 72*** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Nominal position (º)** | **Deviation position 0-0 (”)** | **Deviation position 0-55 (”)** | **Deviation position 0-240 (”)** | **Dev_encoder (”)** | ***Nominal position (º)*** | ***Deviation position 0-0 (”)*** | ***Deviation position 0-55 (”)*** | ***Deviation position 0-240 (”)*** | ***Dev_encoder (”)*** |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | -0,2052 | -0,0144 | -0,1836 | -0,1344 | 5 | -0,1944 | -0,0108 | -0,1836 | -0,1296 |
| 10 | -0,45 | 0,234 | -0,432 | -0,216 | 10 | -0,432 | 0,2376 | -0,4356 | -0,21 |
| 15 | -0,5292 | 0,3672 | -0,5256 | -0,2292 | 15 | -0,5112 | 0,3708 | -0,5256 | -0,222 |
| 20 | -0,4356 | 0,5976 | -0,4536 | -0,0972 | 20 | -0,4356 | 0,6012 | -0,4536 | -0,096 |
| 25 | -0,3456 | 0,5148 | -0,324 | -0,0516 | 25 | -0,324 | 0,5004 | -0,342 | -0,0552 |
| 30 | -0,3528 | 0,3672 | -0,3384 | -0,108 | 30 | -0,3276 | 0,3852 | -0,36 | -0,1008 |
| 35 | -0,288 | 0,1908 | -0,2844 | -0,1272 | 35 | -0,252 | 0,1836 | -0,3096 | -0,126 |
| 40 | -0,2484 | -0,054 | -0,3024 | -0,2016 | 40 | -0,2232 | -0,0468 | -0,3312 | -0,2004 |
| 45 | -0,2664 | 0,0504 | -0,3456 | -0,1872 | 45 | -0,2412 | 0,0468 | -0,4032 | -0,1992 |
| 50 | -0,396 | 0,2052 | -0,4428 | -0,2112 | 50 | -0,3492 | 0,2196 | -0,4644 | -0,198 |
| 55 | -0,4824 | 0,3888 | -0,5832 | -0,2256 | 55 | -0,4356 | 0,3888 | -0,6156 | -0,2208 |
| 60 | -0,4968 | 0,5184 | -0,594 | -0,1908 | 60 | -0,4536 | 0,5436 | -0,6228 | -0,1776 |
| 65 | -0,5976 | 0,3456 | -0,6552 | -0,3024 | 65 | -0,5544 | 0,3708 | -0,6732 | -0,2856 |
| 70 | -0,6228 | 0,3636 | -0,6408 | -0,3 | 70 | -0,5796 | 0,378 | -0,6804 | -0,294 |
| 75 | -0,4464 | 0,4068 | -0,4212 | -0,1536 | 75 | -0,378 | 0,4284 | -0,432 | -0,1272 |
| 80 | -0,3204 | 0,5976 | -0,2844 | -0,0024 | 80 | -0,2556 | 0,6372 | -0,3024 | 0,0264 |
| 85 | -0,1116 | 0,9072 | -0,0432 | 0,2508 | 85 | -0,0504 | 0,918 | -0,0432 | 0,2748 |
| 90 | -0,2736 | 1,044 | -0,1656 | 0,2016 | 90 | -0,2052 | 1,0728 | -0,1584 | 0,2364 |
| 95 | -0,4068 | 1,2636 | -0,288 | 0,1896 | 95 | -0,3456 | 1,26 | -0,288 | 0,2088 |
| 100 | -0,576 | 1,3824 | -0,4212 | 0,1284 | 100 | -0,522 | 1,404 | -0,432 | 0,15 |

| Results taken at forward direction N = 3, L = 3 forward, M = 5, J = 72 | | | | | *Results taken at backward direction N = 3, L = 3 backward, M = 5, J = 72* | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Nominal position (º)** | **Deviation position 0-0 (”)** | **Deviation position 0-55 (”)** | **Deviation position 0-240 (”)** | **Dev_encoder (”)** | ***Nominal position (º)*** | ***Deviation position 0-0 (”)*** | ***Deviation position 0-55 (”)*** | ***Deviation position 0-240 (”)*** | ***Dev_encoder (”)*** |
| 105 | -0,8064 | 1,458 | -0,6516 | 5,11591E-11 | 105 | -0,738 | 1,4688 | -0,6444 | 0,0288 |
| 110 | -0,666 | 1,422 | -0,4752 | 0,0936 | 110 | -0,612 | 1,458 | -0,4716 | 0,1248 |
| 115 | -0,5256 | 1,3356 | -0,2736 | 0,1788 | 115 | -0,4644 | 1,3464 | -0,2916 | 0,1968 |
| 120 | -0,3456 | 1,116 | -0,4248 | 0,1152 | 120 | -0,306 | 1,134 | -0,3024 | 0,1752 |
| 125 | -0,1908 | 0,8748 | -0,288 | 0,132 | 125 | -0,1584 | 0,882 | -0,1404 | 0,1944 |
| 130 | -0,342 | 0,648 | -0,468 | -0,054 | 130 | -0,2844 | 0,6516 | -0,3492 | 0,006 |
| 135 | -0,3132 | 0,4284 | -0,4464 | -0,1104 | 135 | -0,2736 | 0,4464 | -0,3168 | -0,048 |
| 140 | -0,2484 | 0,2304 | -0,3528 | -0,1236 | 140 | -0,1764 | 0,2268 | -0,2376 | -0,0624 |
| 145 | -0,0396 | 0,036 | -0,144 | -0,0492 | 145 | 0,0144 | 0,0396 | -0,036 | 0,006 |
| 150 | 0,2448 | -0,0324 | 0,1584 | 0,1236 | 150 | 0,3168 | -0,0216 | 0,2448 | 0,18 |
| 155 | 0,3672 | 0,018 | 0,2844 | 0,2232 | 155 | 0,4356 | 0,0288 | 0,3636 | 0,276 |
| 160 | 0,5472 | 0,2952 | 0,5436 | 0,462 | 160 | 0,6084 | 0,2952 | 0,6192 | 0,5076 |
| 165 | 0,6228 | 0,5256 | 0,6624 | 0,6036 | 165 | 0,7092 | 0,5256 | 0,7092 | 0,648 |
| 170 | 0,684 | 0,486 | 0,7092 | 0,6264 | 170 | 0,756 | 0,4644 | 0,7848 | 0,6684 |
| 175 | 0,6408 | 0,2592 | 0,702 | 0,534 | 175 | 0,7308 | 0,2664 | 0,7668 | 0,588 |
| 180 | 0,4968 | 0,0648 | 0,5832 | 0,3816 | 180 | 0,5868 | 0,0576 | 0,6264 | 0,4236 |
| 185 | 0,2484 | 0,0324 | 0,3204 | 0,2004 | 185 | 0,3348 | 0,0144 | 0,4068 | 0,252 |
| 190 | 0 | 0 | 0,0792 | 0,0264 | 190 | 0,1116 | 0,018 | 0,1548 | 0,0948 |
| 195 | -0,2052 | 0,0252 | -0,1512 | -0,1104 | 195 | -0,1044 | 0,0324 | -0,108 | -0,06 |
| 200 | -0,3996 | -0,054 | -0,3492 | -0,2676 | 200 | -0,2916 | -0,09 | -0,3204 | -0,234 |
| 205 | -0,6156 | -0,2844 | -0,5868 | -0,4956 | 205 | -0,5112 | -0,2988 | -0,5256 | -0,4452 |
| 210 | -0,8136 | -0,5328 | -0,774 | -0,7068 | 210 | -0,6876 | -0,5652 | -0,7488 | -0,6672 |
| 215 | -0,8352 | -0,7272 | -0,81 | -0,7908 | 215 | -0,7344 | -0,7524 | -0,7812 | -0,756 |
| 220 | -0,7488 | -0,6012 | -0,7956 | -0,7152 | 220 | -0,6084 | -0,6336 | -0,738 | -0,66 |
| 225 | -0,4752 | -0,1188 | -0,5652 | -0,3864 | 225 | -0,342 | -0,1476 | -0,504 | -0,3312 |

| Results taken at forward direction $N = 3$, $L = 3$ forward, $M = 5$, $J = 72$ | | | | | ***Results taken at backward direction*** $N = 3$, $L = 3$ backward, $M = 5$, $J = 72$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Nominal position (º)** | **Deviation position 0-0 (”)** | **Deviation position 0-55 (”)** | **Deviation position 0-240 (”)** | **Dev_encoder (”)** | ***Nominal position* (º)** | ***Deviation position* 0-0 (”)** | ***Deviation position* 0-55 (”)** | ***Deviation position 0-240* (”)** | ***Dev_encoder* (”)** |
| 230 | -0,2088 | 0,5184 | -0,3132 | -0,0012 | 230 | -0,0792 | 0,486 | -0,252 | 0,0516 |
| 235 | -0,2304 | 1,098 | -0,4032 | 0,1548 | 235 | -0,1548 | 1,0692 | -0,3204 | 0,198 |
| 240 | -0,4464 | 1,2888 | -0,5976 | 0,0816 | 240 | -0,3744 | 1,3068 | -0,5148 | 0,1392 |
| 245 | -0,6516 | 1,3968 | -0,774 | -0,0096 | 245 | -0,5472 | 1,4112 | -0,7236 | 0,0468 |
| 250 | -0,7272 | 1,2312 | -0,864 | -0,12 | 250 | -0,6372 | 1,2528 | -0,7884 | -0,0576 |
| 255 | -0,8064 | 1,1448 | -0,9108 | -0,1908 | 255 | -0,7128 | 1,1664 | -0,8316 | -0,126 |
| 260 | -0,792 | 1,0116 | -0,8712 | -0,2172 | 260 | -0,702 | 1,044 | -0,7416 | -0,1332 |
| 265 | -0,9072 | 0,918 | -0,954 | -0,3144 | 265 | -0,7956 | 0,9288 | -0,8532 | -0,24 |
| 270 | -1,1808 | 0,8748 | -1,242 | -0,516 | 270 | -1,0836 | 0,8964 | -1,134 | -0,4404 |
| 275 | -1,4472 | 0,7812 | -1,4976 | -0,7212 | 275 | -1,35 | 0,7956 | -1,3968 | -0,6504 |
| 280 | -1,6668 | 0,648 | -1,71 | -0,9096 | 280 | -1,5732 | 0,648 | -1,6236 | -0,8496 |
| 285 | -1,5336 | 0,5544 | -1,6092 | -0,8628 | 285 | -1,4508 | 0,5544 | -1,5408 | -0,8124 |
| 290 | -1,0296 | 0,7128 | -1,134 | -0,4836 | 290 | -0,9216 | 0,738 | -1,0656 | -0,4164 |
| 295 | -0,4536 | 0,6948 | -0,5436 | -0,1008 | 295 | -0,3384 | 0,7092 | -0,4644 | -0,0312 |
| 300 | 0,0936 | 0,5148 | 0,0216 | 0,21 | 300 | 0,2088 | 0,5364 | 0,0756 | 0,2736 |
| 305 | 0,5688 | 0,27 | 0,4392 | 0,426 | 305 | 0,6624 | 0,2808 | 0,4932 | 0,4788 |
| 310 | 0,7452 | 0,1656 | 0,5724 | 0,4944 | 310 | 0,8532 | 0,1836 | 0,6192 | 0,552 |
| 315 | 0,594 | 0,2016 | 0,4248 | 0,4068 | 315 | 0,6876 | 0,2124 | 0,468 | 0,456 |
| 320 | 0,54 | 0,324 | 0,3564 | 0,4068 | 320 | 0,6336 | 0,324 | 0,3924 | 0,45 |
| 325 | 0,4284 | 0,3348 | 0,2304 | 0,3312 | 325 | 0,54 | 0,3384 | 0,2664 | 0,3816 |
| 330 | 0,3456 | 0,3636 | 0,1512 | 0,2868 | 330 | 0,4608 | 0,396 | 0,18 | 0,3456 |
| 335 | 0,2844 | 0,3564 | 0,1188 | 0,2532 | 335 | 0,4104 | 0,3636 | 0,1476 | 0,3072 |
| 340 | 0,252 | 0,306 | 0,0684 | 0,2088 | 340 | 0,3672 | 0,3132 | 0,072 | 0,2508 |
| 345 | 0,0756 | 0,1872 | -0,09 | 0,0576 | 345 | 0,2052 | 0,1764 | -0,0756 | 0,102 |
| 350 | 0,018 | 0,0756 | -0,144 | -0,0168 | 350 | 0,1332 | 0,0828 | -0,126 | 0,03 |
| 355 | 0,2124 | 0,0396 | 0,0324 | 0,0948 | 355 | 0,3348 | 0,072 | 0,054 | 0,1536 |




Note A1: The laboratories shall decide the number of sets and relative positions considering their own angle reference system and set-up.

## 2 APPENDIX B: Example of an Uncertainty budget for calibration of encoders

| Description of input quantity $x_i$ | Symbol for $x_i$ | Type A or B | Distri-bution | Standard measurement uncertainty $u(x_i)$ of input quantity (arcseconds) | Degrees of freedom $v_i$ | Sensitivity coefficient $c_i$ | Standard measurement uncertainty contribution $u_i$ (arcseconds) |
|---|---|---|---|---|---|---|---|
| Standard deviation of the mean value | $\Delta\alpha$ | A | N | $u_A$ | 89 | 1 | 0.022 |
| Resolution of the test encoder | $a_{CAL}$ | B | R | $u_{resCAL}$ | ∞ | 1 | 0.021 |
| Resolution of the reference encoder | $a_{REF}$ | B | R | $u_{resREF}$ | ∞ | 1 | 0.010 |
| Reference encoder calibration | $U_{cerREF}$ | B | R | $u_{cerREF}$ | ∞ | 1 | 0.050 |
| Interpolation error of the reference encoder | $e_{intREF}$ | B | R | $u_{intREF}$ | ∞ | 1 | 0.029 |
| Influence of the residual angle between axes of rotation | $e_{aba}$ | B | R | $u_{aba}$ | ∞ | 1 | 0.058 |
| Combined standard measurement uncertainty $u_c$ = 0.09<br>Coverage factor $k$ for a coverage probability of 95% = 2<br>Effective degrees of freedom $v_{eff}$ = ∞ ( > 5E9) | | | | | | | |

This uncertainty budget is the evaluation of the uncertainty in the calibration of a commercial encoder by comparison against a standard turntable with integrated reference encoder. Measurements were taken every 5° till covering the full 360° range.

The uncertainty contribution of eccentricity and inclination can be evaluated from the residuals obtained by comparing the set of measured errors of the encoder in each interval with the reconstructed error of the encoder in the same interval.

## 3 APPENDIX C: Explanations for the effects due to inclination angle and eccentricity between reference and test encoders

When two angle encoders are mounted for comparison, the following adjustment errors are possible. The user may measure or estimate these errors and calculate the resultant angular error in order to include it in the uncertainty budget.

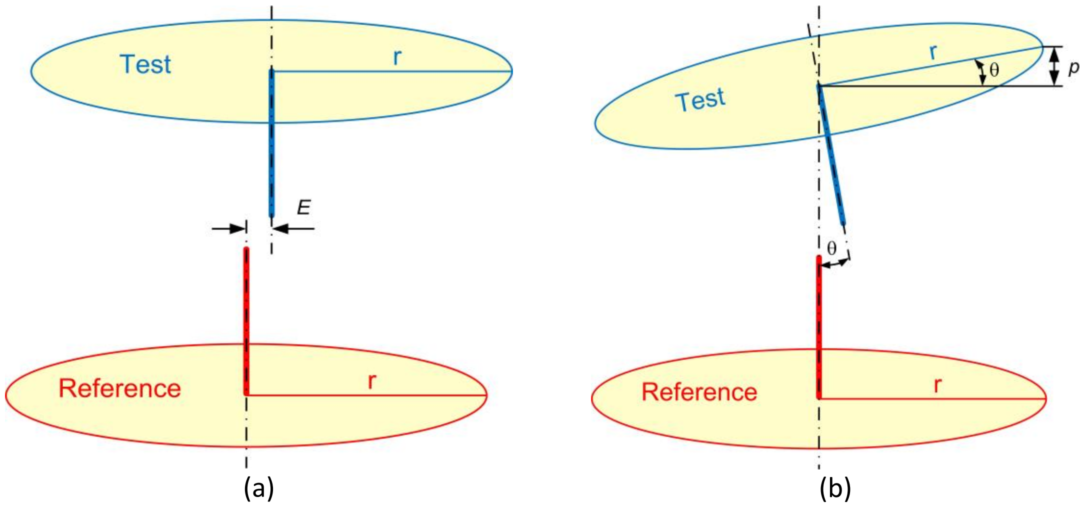



Figure C.1.1 Explanation of the effects due to the eccentricity (a) and inclination angle (b) between the reference and the test encoder.

### a) The influence of the eccentricity between test and reference angle encoder axes

In the event of eccentricity $E$ between test and reference angle measurement system, systematic error effects occur with the following relationship (C-1.1):

$$e_E = \pm 2 \times \left(\frac{E}{r}\right) \times \sin\left(\frac{\alpha}{2}\right) \times \left(\frac{180}{\Pi}\right) \qquad \text{(C-1.1)}$$

(α = 180° for extreme case)

**b) The influence of the angle between the axes of rotation (inclination angle between test and reference angle encoder axes)**

In the event of non-parallelism of the measurement planes between test and reference angle measurement system with a lateral run-out ± *p*, systematic error effects occur with the following relationship (C-1.2):

$$e_p = \pm \frac{1}{8} \times \left(2 \times \frac{p}{r}\right)^2 \times \sin\alpha \times \left(\frac{180}{\Pi}\right) \qquad \text{(C-1.2)}$$

(α = 90° for extreme case)

where,

α : Angle to be measured
*r* : Effective radius of the angle measurement
*E* : Eccentricity
*p* : Lateral run-out (non-parallelism) of the two measurement planes of the reference and test.
$e_E$: Angular error as ‘arcseconds’ resulting from eccentricity *E*
$e_p$: Angular error as ‘arcseconds’ resulting from (non-parallelism) lateral run-out *p*.

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