EURAMET Project 1213

Calibration Factor of a 2.92 mm Thermoelectric Power Sensor up to 40 GHz

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Abstract

This report summarises the results of an interlaboratory measurement comparison on a commercial thermoelectric power sensor. Two national metrology institutes participated in the comparison where the calibration factor and the input reflection coefficient were the measurands. Both laboratories applied a direct comparison setup using reference power sensors that were traced to different primary standards.

Contents

1 Introduction

During the last decade, the importance of thermoelectric RF power sensors used as transfer standards has continuously increased. This is due to the fact that they are well-matched devices and offer properties like high efficiency, high linearity, broadband behaviour, and fast response compared to thermistor power sensors. Furthermore, thermistor power sensors are practically unavailable even on the second-hand market, but they are still used as standards to obtain traceability to SI units. To establish a link between both sensor types and to obtain traceablity for the quantity "high frequency power", the direct comparison method [1] is established as a fast, accurate, and broadband method both in national metrology institutes and in accredited calibration laboratories.

In this bilateral comparison, the calibration factor of a commercial thermoelectric power sensor for the 2.92 mm coaxial line system has been determined by utilising direct comparison setups in both laboratories. Since the sensor reflection coefficient is a necessary quantity to determine the calibration factor it is also included in the comparison.

2 Participants and Schedule

Table 1 lists the participants of the bilateral comparison. The duties were shared among the participants where SP acted as the pilot laboratory by providing the travelling standard, performing the initial and final measurement and analysing the data while PTB compiled the report.

Table 1: Participants of the comparison.

Acronym	Laboratory
SP	SP Technical Research Institute of Sweden Box 857, SE-501 15 Borås, Sweden contact: Klas Yhland klas.yhland@sp.se
PTB	Physikalisch-Technische Bundesanstalt Bundesallee 100, D-38116 Braunschweig contact: Rolf Judaschke rolf.judaschke@ptb.de

Table 2 lists the sequence of measurements performed by the laboratories. The last column indicates the overall time elapsed from the initial measurement.

Table 2: Sequence of measurements during the comparison.

Laboratory	Action	Identifier	Result	Date of measurement	Time delay days
SP	Initial measurement	SP1	Cal certificate	16.06.2009	0
PTB	Measurement 1	PTB1	Data transfer	21.10.2009	127
PTB	Measurement 2	PTB2	Data transfer	25.08.2010	435
SP	Final measurement	SP2	Cal certificate	26.08.2011	801

The methods of calibration that were applied by the participants are described in section B.1.1 and B.2.1.

3 Travelling Standard and Measurements

The travelling standard was a thermoelectric power sensor type Rohde & Schwarz NRV-Z55, SN100014, with a PC2.92 mm male connector, frequency range DC to 40 GHz. Both laboratories used their own corresponding NRVD power meter for the sensor readout.

The comparison included the following measurands:

- Relative calibration factor η with respect to 50 MHz at an incident power level of 1 mW in the frequency range 50 MHz to 40 GHz
- Complex input reflection coefficient Γ of the power sensor

The relative calibration factor η is calculated from

$$\eta(f) = \frac{\eta_{\text{abs}}(f)}{\eta_{\text{abs}}(50\,\text{MHz})} \tag{1}$$

where the absolute calibration factor is defined as

$$\eta_{\text{abs}}(f) = \frac{P_{\text{ind}}(f)}{P_{\text{inc}}(f)} \,. \tag{2}$$

In (2), P_{ind} and P_{inc} denote the high frequency power indicated by the power meter and the incident power at the sensor input plane, respectively.

The measurement results were documented and transferred in the form of excel spreadsheets and in a GUM [2] compliant measurement uncertainty budget.

Tables of the electronic data submitted by the participants are listed in Appendix A. Information about the measurement setups as well as the uncertainty evaluation can be found in Appendix B.

3.1 Stability of the Travelling Standard

The initial and final measurements, denoted SP1 and SP2, were performed by SP according to Table 2. The cross check between SP1 and SP2 in Appendix C indicates a minor change in the calibration factor of the sensor during the comparison.

Since it is unknown when the change in the calibration factor occured an average between measurements SP1 and SP2 (denoted SP) was used in the comparison with the two measurements from PTB (denoted PTB1 and PTB2). The difference between SP1 and SP2 is used as an uncertainty contribution in the comparison. The same approach was used for the comparison of the reflection coefficient.

4 Data Analysis

The analysis of the measurement data covers the averaged measurements of SP (denoted SP) and two measurements of PTB (denoted as PTB1 and PTB2) where different adapter characterisation and source reflection measurement methods were applied in PTB1 and PTB2, respectively. All numerical data are listed in Appendix A.

The calibration factor normalised error E_n was calculated to characterise the difference between the results of the participants. It is defined as

$$E_n = \frac{\eta_{\text{SP}} - \eta_{\text{PTB}}}{2\sqrt{u(\eta_{\text{SP}})^2 + u(\eta_{\text{PTB}})^2}} = \frac{\text{error}}{U(\text{error})} \tag{3}$$

with the quantities

η_{SP}	calibration factor of SP,
η_{PTB}	calibration factor of PTB,
$u(\eta_{\text{SP}})$	standard uncertainty of SP,
$u(\eta_{\text{PTB}})$	standard uncertainty of PTB,
$\eta_{\text{SP}} - \eta_{\text{PTB}}$	error between SP and PTB calibration factors.

$|E_n| < 1$ indicates an agreement between the participants at the 95 % level.

The reflection coefficient normalised error E_n was calculated to characterise the difference between the results of the participants. It is defined as

$$E_n = \frac{|\Gamma_{\text{SP}} - \Gamma_{\text{PTB}}|}{2.45\sqrt{u(\Gamma_{\text{SP}})^2 + u(\Gamma_{\text{PTB}})^2}} = \frac{\text{error}}{U(\text{error})} \tag{4}$$

with the quantities

Γ_{SP}	reflection coefficient of SP,
Γ_{PTB}	reflection coefficient of PTB,
$u(\Gamma_{\text{SP}})$	standard uncertainty of SP,
$u(\Gamma_{\text{PTB}})$	standard uncertainty of PTB,
$\lvert\Gamma_{\text{SP}} - \Gamma_{\text{PTB}}\rvert$	error between SP and PTB reflection coefficients.

$|E_n| < 1$ indicates an agreement between the participants at the 95 % level.

4.1 Results

For each calibration factor comparison (with respect to the performed measurements), the results are summarised in three diagrams. The relative calibration factors are shown in Fig. 1 and Fig. 4. The error between the calibration factors and the combined uncertainty is shown in Fig. 2 and Fig. 5. The normalised error is shown in Fig. 3 and Fig. 6. Fig. 7 shows the expanded uncertainties for the individual measurements.

The comparison between SP and the first measurement by PTB results in normalised errors $|E_n| < 1$ at all frequencies. However, at one single frequency the normalised error is very close to one, $|E_n| = 0.98$ @ 37 GHz. In the second measurement by PTB different adapter characterisation and source reflection measurement methods where used. In this case $|E_n| < 0.6$ over the entire frequency range which indicates good overall agreement between the partcipants.

For the reflection coefficient comparison, the results are summarised in four diagrams. The reflection coefficients are shown in Fig. 8. The error between the reflection coefficients and the combined uncertainty is shown in Fig. 9. The normalised error is shown in Fig. 10. Fig. 11 shows the expanded uncertainties for the individual measurements.

The reflection coefficient comparison between SP and the first measurement by PTB results in normalised errors $|E_n| < 1$ at all frequencies.

4.1.1 Calibration Factor SP - PTB1

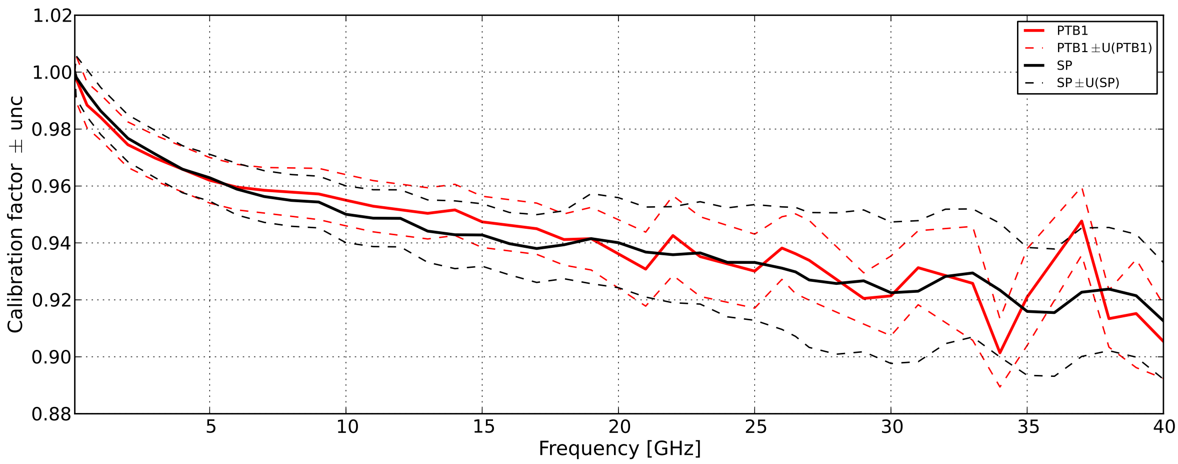

Fig 1: Calibration factors η_{SP} and η_{PTB1} and their expanded uncertainties (k=2).

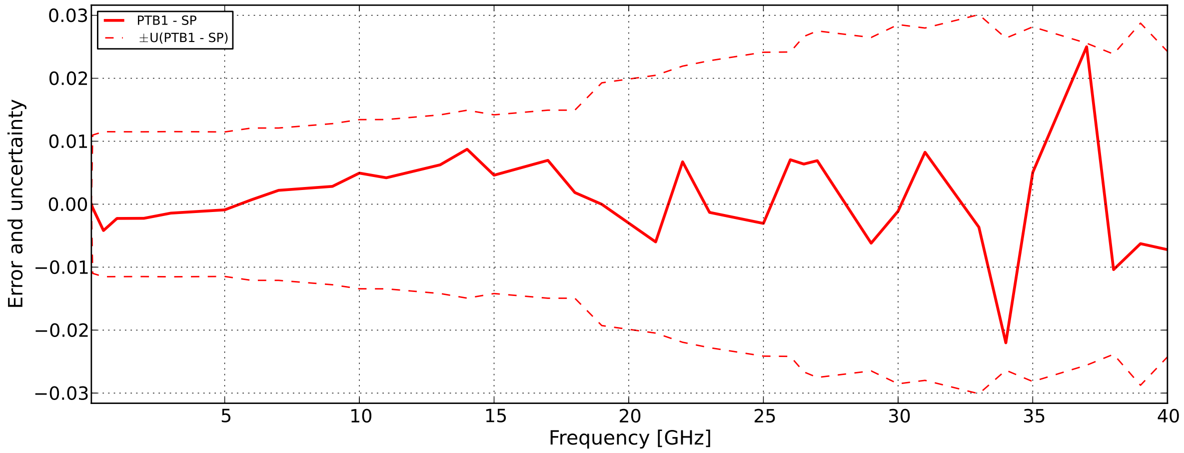

Fig 2: Error between η_{SP} and η_{PTB1} and the combined expanded uncertainty (k=2).

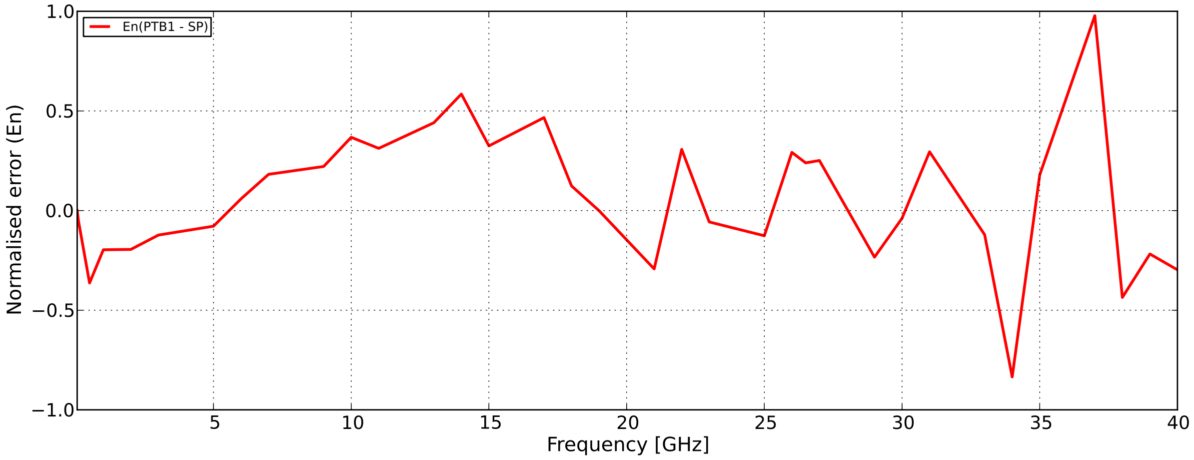

Fig 3: Normalised error between η_{SP} and η_{PTB1}.

4.1.2 Calibration Factor SP - PTB2

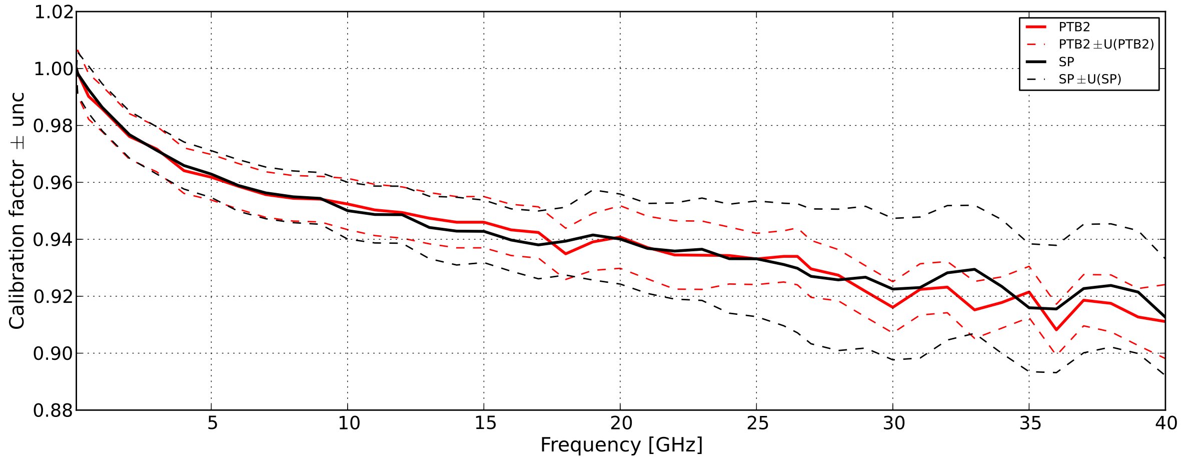

Fig 4: Calibration factors η_{SP} and η_{PTB2} and their expanded uncertainties (k=2).

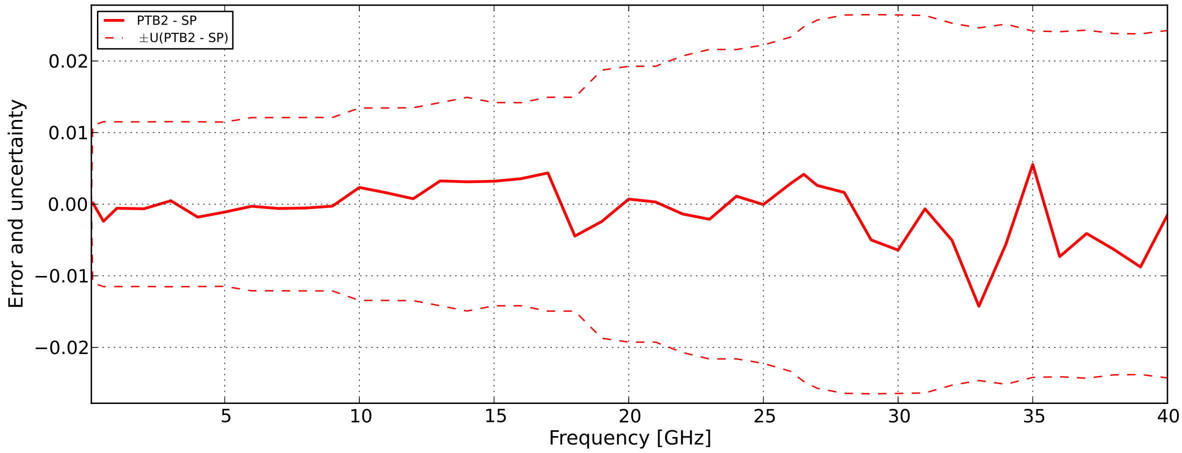

Fig 5: Error between η_{SP} and η_{PTB2} and the combined expanded uncertainty (k=2).

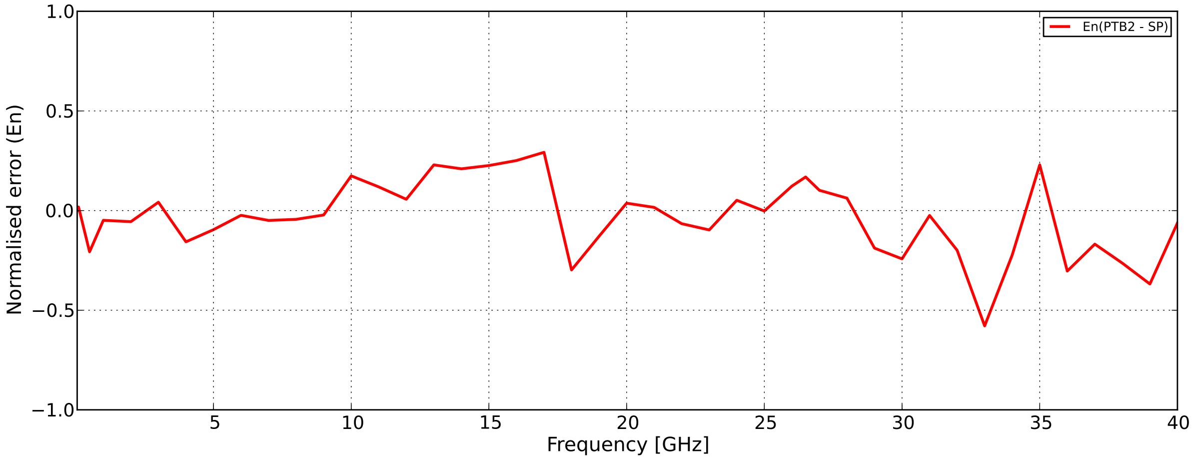

Fig 6: Normalised error between η_{SP} and η_{PTB2}.

4.1.3 Comparison of Measurement Uncertainties

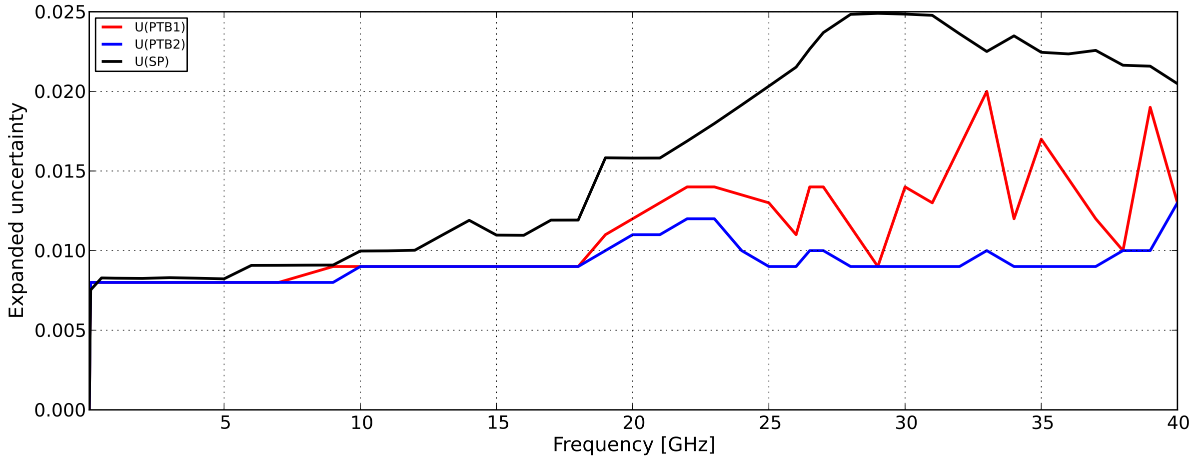

Fig 7: Expanded uncertainties ($k = 2$) of η_{SP}, η_{PTB1}, and η_{PTB2}.

4.1.4 Reflection coefficient SP - PTB1

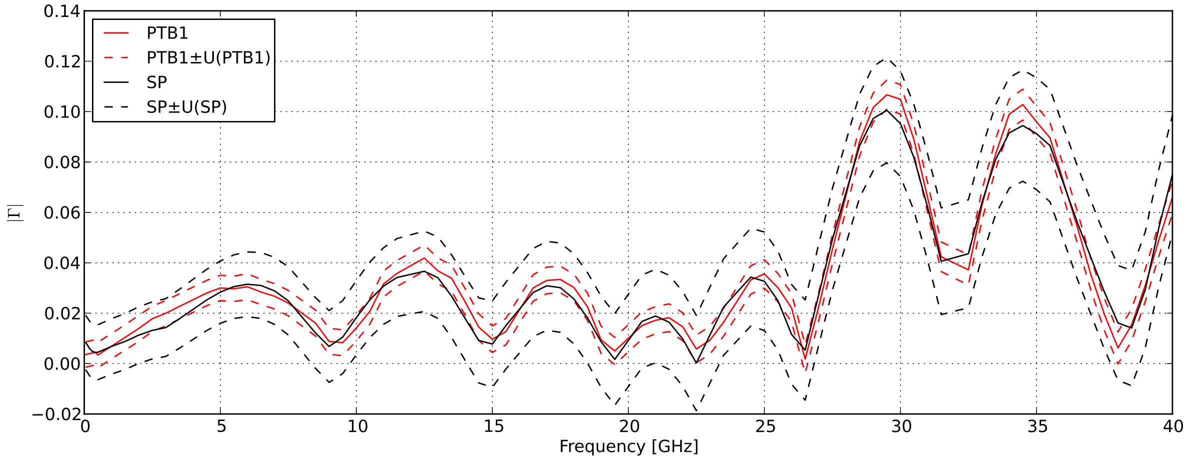

Fig 8: Reflection coefficients $|\Gamma_{\mathrm{SP}}|$ and $|\Gamma_{\mathrm{PTB1}}|$ and their expanded uncertainties (k=2.45).

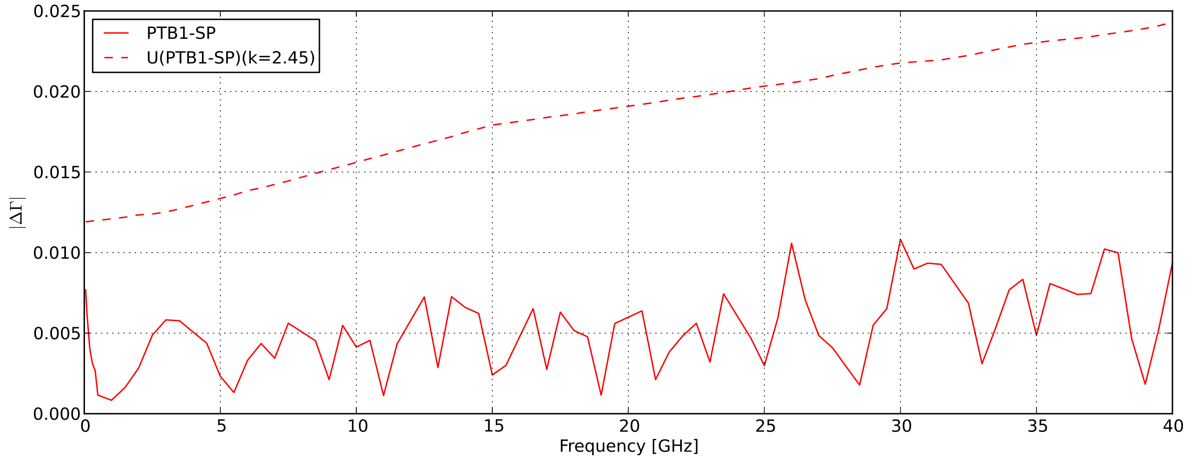

Fig 9: Error between Γ_{SP} and Γ_{PTB1} and the combined expanded uncertainty (k=2.45).

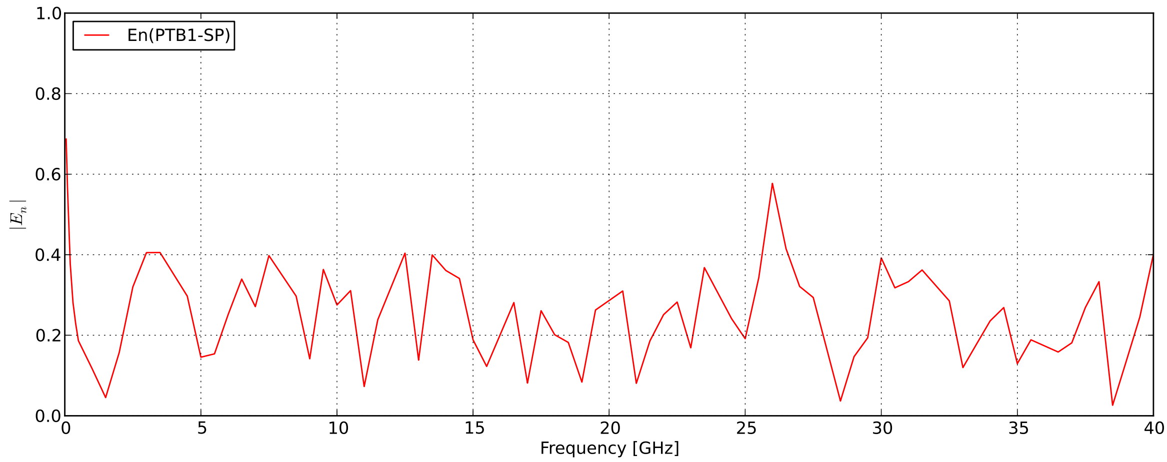

Fig 10: Normalised error between Γ_{SP} and Γ_{PTB1}.

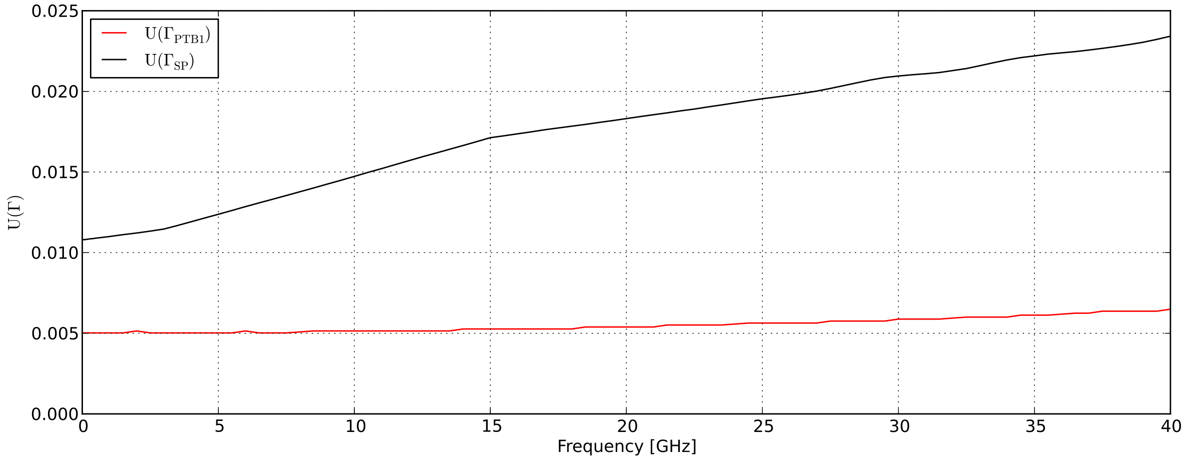

Fig 11: Expanded uncertainties (k=2.45) of Γ_{SP} and Γ_{PTB1}.

5 Conclusions

A bilateral comparison for the calibration factor (normalised frequency response) of a thermocouple power sensor was performed. Both participants used a direct comparison setup with a set of thermistor power sensors as standards (PTB) and a thermocouple reference standard (SP), respectively. Acceptable agreement was achieved for the first comparison loop (SP - PTB1), while the second loop (SP - PTB2) resulted in good agreement.

For the second loop, PTB improved both the effective source match determination of the generator system and the characterisation of the waveguide-to-coaxial adapters between the used waveguide standards and the reference plane. Since the available waveguide reference standards are poorly matched especially at higher frequencies, a careful determination of both generator source match and adapters is essential.

A Measurement Data

A.1 Calibration factor data SP - PTB1

Table 3: Measurement data SP - PTB1.

Frequency GHz	E_n	Error	U(Error) k=2	η_{SP}	$u(\eta_{SP})$	η_{PTB1}	$u(\eta_{PTB1})$
0.05	0	0	0	1	0	1	0
0.10	−0.05	−0.001	0.011	0.9982	0.0038	0.9976	0.0040
0.50	−0.36	−0.004	0.012	0.9926	0.0041	0.9884	0.0040
1.00	−0.20	−0.002	0.012	0.9864	0.0041	0.9841	0.0040
2.00	−0.19	−0.002	0.011	0.9767	0.0041	0.9745	0.0040
3.00	−0.12	−0.001	0.012	0.9712	0.0042	0.9698	0.0040
5.00	−0.08	−0.001	0.011	0.9629	0.0041	0.9620	0.0040
6.00	0.06	0.001	0.012	0.9589	0.0045	0.9596	0.0040
7.00	0.18	0.002	0.012	0.9563	0.0045	0.9585	0.0040
9.00	0.22	0.003	0.013	0.9544	0.0045	0.9572	0.0045
10.00	0.37	0.005	0.013	0.9501	0.0050	0.9550	0.0045
11.00	0.31	0.004	0.013	0.9487	0.0050	0.9529	0.0045
13.00	0.44	0.006	0.014	0.9441	0.0055	0.9504	0.0045
14.00	0.58	0.009	0.015	0.9429	0.0060	0.9516	0.0045
15.00	0.32	0.005	0.014	0.9428	0.0055	0.9474	0.0045
17.00	0.47	0.007	0.015	0.9380	0.0060	0.9450	0.0045
18.00	0.12	0.002	0.015	0.9394	0.0060	0.9412	0.0045
19.00	−0.00	−0.000	0.019	0.9415	0.0079	0.9415	0.0055
21.00	−0.29	−0.006	0.020	0.9368	0.0079	0.9308	0.0065
22.00	0.31	0.007	0.022	0.9359	0.0084	0.9426	0.0070
23.00	−0.06	−0.001	0.023	0.9365	0.0090	0.9352	0.0070
25.00	−0.13	−0.003	0.024	0.9332	0.0102	0.9301	0.0065
26.00	0.29	0.007	0.024	0.9311	0.0108	0.9382	0.0055
26.50	0.24	0.006	0.027	0.9298	0.0113	0.9362	0.0070
27.00	0.25	0.007	0.028	0.9270	0.0118	0.9339	0.0070
29.00	−0.23	−0.006	0.026	0.9267	0.0124	0.9205	0.0045
30.00	−0.04	−0.001	0.029	0.9225	0.0124	0.9214	0.0070
31.00	0.29	0.008	0.028	0.9231	0.0124	0.9313	0.0065
33.00	−0.12	−0.004	0.030	0.9295	0.0113	0.9258	0.0100
34.00	−0.83	−0.022	0.026	0.9234	0.0117	0.9014	0.0060
35.00	0.18	0.005	0.028	0.9160	0.0112	0.9210	0.0085
37.00	0.98	0.025	0.026	0.9227	0.0113	0.9477	0.0060
38.00	−0.44	−0.010	0.024	0.9238	0.0108	0.9134	0.0050
39.00	−0.22	−0.006	0.029	0.9215	0.0108	0.9152	0.0095
40.00	−0.30	−0.007	0.024	0.9126	0.0102	0.9054	0.0065

A.2 Calibration factor data SP - PTB2

Table 4: Measurement data SP - PTB2.

Frequency GHz	E_n	Error	U(Error) k=2	η_{SP}	$u(\eta_{\mathrm{SP}})$	η_{PTB1}	$u(\eta_{\mathrm{PTB1}})$
0.05	0	0	0	1	0	1	0
0.10	0.02	0.000	0.011	0.9982	0.0038	0.9984	0.0040
0.50	−0.21	−0.002	0.012	0.9926	0.0041	0.9902	0.0040
1.00	−0.05	−0.001	0.012	0.9864	0.0041	0.9858	0.0040
2.00	−0.06	−0.001	0.011	0.9767	0.0041	0.9761	0.0040
3.00	0.04	0.000	0.012	0.9712	0.0042	0.9717	0.0040
4.00	−0.16	−0.002	0.012	0.9659	0.0041	0.9641	0.0040
5.00	−0.10	−0.001	0.011	0.9629	0.0041	0.9618	0.0040
6.00	−0.02	−0.000	0.012	0.9589	0.0045	0.9586	0.0040
7.00	−0.05	−0.001	0.012	0.9563	0.0045	0.9557	0.0040
8.00	−0.04	−0.001	0.012	0.9549	0.0045	0.9544	0.0040
9.00	−0.02	−0.000	0.012	0.9544	0.0045	0.9541	0.0040
10.00	0.17	0.002	0.013	0.9501	0.0050	0.9524	0.0045
11.00	0.12	0.002	0.013	0.9487	0.0050	0.9503	0.0045
12.00	0.06	0.001	0.013	0.9486	0.0050	0.9494	0.0045
13.00	0.23	0.003	0.014	0.9441	0.0055	0.9474	0.0045
14.00	0.21	0.003	0.015	0.9429	0.0060	0.9460	0.0045
15.00	0.23	0.003	0.014	0.9428	0.0055	0.9460	0.0045
16.00	0.25	0.004	0.014	0.9397	0.0055	0.9433	0.0045
17.00	0.29	0.004	0.015	0.9380	0.0060	0.9424	0.0045
18.00	−0.30	−0.004	0.015	0.9394	0.0060	0.9349	0.0045
19.00	−0.13	−0.002	0.019	0.9415	0.0079	0.9391	0.0050
20.00	0.04	0.001	0.019	0.9401	0.0079	0.9408	0.0055
21.00	0.02	0.000	0.019	0.9368	0.0079	0.9371	0.0055
22.00	−0.07	−0.001	0.021	0.9359	0.0084	0.9345	0.0060
23.00	−0.10	−0.002	0.022	0.9365	0.0090	0.9344	0.0060
24.00	0.05	0.001	0.022	0.9332	0.0096	0.9343	0.0050
25.00	−0.00	−0.000	0.022	0.9332	0.0102	0.9331	0.0045
26.00	0.12	0.003	0.023	0.9311	0.0108	0.9340	0.0045
26.50	0.17	0.004	0.025	0.9298	0.0113	0.9340	0.0050
27.00	0.10	0.003	0.026	0.9270	0.0118	0.9296	0.0050
28.00	0.06	0.002	0.026	0.9257	0.0124	0.9274	0.0045
29.00	−0.19	−0.005	0.026	0.9267	0.0124	0.9217	0.0045
30.00	−0.24	−0.006	0.026	0.9225	0.0124	0.9161	0.0045
31.00	−0.02	−0.001	0.026	0.9231	0.0124	0.9224	0.0045
32.00	−0.20	−0.005	0.025	0.9282	0.0118	0.9232	0.0045
33.00	−0.58	−0.014	0.025	0.9295	0.0113	0.9152	0.0050
34.00	−0.22	−0.006	0.025	0.9234	0.0117	0.9178	0.0045
35.00	0.23	0.006	0.024	0.9160	0.0112	0.9215	0.0045
36.00	−0.30	−0.007	0.024	0.9155	0.0112	0.9082	0.0045
37.00	−0.17	−0.004	0.024	0.9227	0.0113	0.9186	0.0045
38.00	−0.26	−0.006	0.024	0.9238	0.0108	0.9175	0.0050
39.00	−0.37	−0.009	0.024	0.9215	0.0108	0.9127	0.0050
40.00	−0.06	−0.002	0.024	0.9126	0.0102	0.9111	0.0065

A.3 Reflection coefficicent data SP - PTB1

Table 5: Measurement data SP - PTB1.

Frequency GHz	E_n	Error	U(Error) k=2.45	Γ_{SP} Re	Γ_{SP} Im	$U(\Gamma_{SP})$ k=2.45	Γ_{PTB1} Re	Γ_{PTB1} Im	$U(\Gamma_{PTB1})$ k=2.45
0.05	0.687	0.008	0.012	−0.0061	0.0057	0.0108	−0.0033	−0.0015	0.0050
0.10	0.564	0.006	0.012	−0.0057	0.0051	0.0108	−0.0036	−0.0008	0.0050
0.50	0.186	0.001	0.012	−0.0028	0.0036	0.0109	−0.0018	0.0029	0.0050
1.00	0.118	0.001	0.012	−0.0026	0.0063	0.0110	−0.0019	0.0066	0.0050
2.00	0.157	0.003	0.012	0.0077	0.0082	0.0112	0.0104	0.0093	0.0051
3.00	0.405	0.006	0.013	0.0136	−0.0046	0.0115	0.0194	−0.0049	0.0050
5.00	0.146	0.002	0.013	−0.0169	−0.0228	0.0124	−0.0165	−0.0250	0.0050
6.00	0.251	0.003	0.014	−0.0310	−0.0058	0.0129	−0.0304	−0.0025	0.0051
7.00	0.271	0.003	0.014	−0.0245	0.0152	0.0133	−0.0212	0.0162	0.0050
9.00	0.142	0.002	0.015	0.0068	−0.0004	0.0142	0.0088	0.0003	0.0051
10.00	0.275	0.004	0.016	−0.0106	−0.0148	0.0147	−0.0080	−0.0116	0.0051
11.00	0.073	0.001	0.016	−0.0307	−0.0018	0.0152	−0.0315	−0.0026	0.0051
13.00	0.138	0.003	0.017	−0.0023	0.0340	0.0162	−0.0015	0.0368	0.0051
14.00	0.361	0.007	0.017	0.0131	0.0122	0.0166	0.0175	0.0172	0.0053
15.00	0.189	0.002	0.018	−0.0007	−0.0077	0.0171	0.0007	−0.0097	0.0053
17.00	0.081	0.003	0.018	−0.0224	0.0212	0.0176	−0.0227	0.0240	0.0053
18.00	0.201	0.005	0.019	−0.0015	0.0256	0.0178	0.0009	0.0302	0.0053
19.00	0.084	0.001	0.019	0.0053	0.0068	0.0181	0.0050	0.0079	0.0054
21.00	0.081	0.002	0.019	−0.0136	0.0132	0.0186	−0.0115	0.0128	0.0054
22.00	0.251	0.005	0.020	−0.0001	0.0098	0.0188	−0.0001	0.0146	0.0055
23.00	0.169	0.003	0.020	−0.0079	−0.0082	0.0190	−0.0079	−0.0050	0.0055
25.00	0.191	0.003	0.020	−0.0240	0.0222	0.0195	−0.0257	0.0246	0.0056
26.00	0.577	0.011	0.021	−0.0007	0.0116	0.0198	0.0001	0.0221	0.0056
26.50	0.415	0.007	0.021	−0.0022	−0.0049	0.0199	0.0013	0.0013	0.0056
27.00	0.321	0.005	0.021	−0.0151	−0.0229	0.0200	−0.0117	−0.0194	0.0056
29.00	0.147	0.005	0.022	−0.0778	0.0583	0.0207	−0.0833	0.0581	0.0058
30.00	0.392	0.011	0.022	−0.0075	0.0950	0.0210	−0.0133	0.1040	0.0059
31.00	0.333	0.009	0.022	0.0502	0.0365	0.0211	0.0477	0.0454	0.0059
33.00	0.120	0.003	0.022	−0.0638	0.0045	0.0216	−0.0625	0.0073	0.0060
34.00	0.236	0.008	0.023	−0.0392	0.0827	0.0220	−0.0406	0.0903	0.0060
35.00	0.130	0.005	0.023	0.0448	0.0795	0.0222	0.0461	0.0842	0.0061
37.00	0.181	0.007	0.023	0.0207	−0.0367	0.0226	0.0198	−0.0293	0.0062
38.00	0.333	0.010	0.024	0.0015	−0.0160	0.0228	−0.0002	−0.0062	0.0064
39.00	0.135	0.002	0.024	0.0172	−0.0237	0.0230	0.0186	−0.0248	0.0064
40.00	0.401	0.009	0.024	−0.0171	−0.0731	0.0234	−0.0171	−0.0638	0.0065

B Addional Information by the Participants

B.1 SP

B.1.1 Method of calibration

SP uses the direct calibration setup [1] shown in Fig. 12. The volt meter, the standard power sensor, and the DUT (device under test) are alternately connected to the power splitter. Their readout is taken simultaneously with the monitoring power sensor.

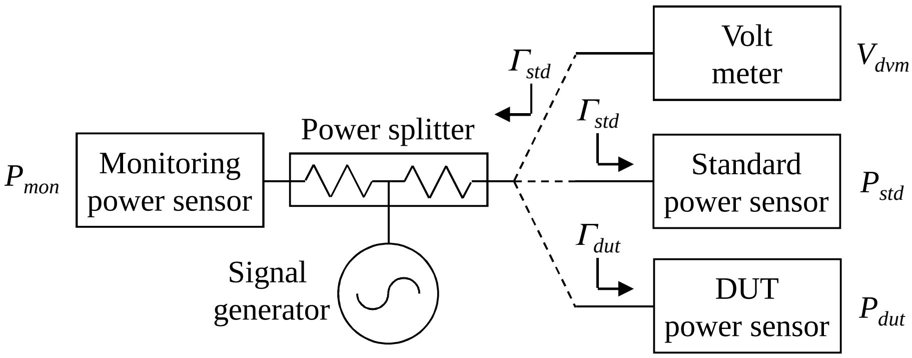

Fig 12: The calibration setup used at SP.

The frequency dependent calibration factor of the DUT is traced to the standard sensor. The absolute power level in the setup is traced to the volt meter. The reflection coefficient of the standard sensor Γ_{std} and the DUT Γ_{dut} are measured with a VNA. The equivalent source reflection coefficient Γ_{gen} of the power splitter is measured with a VNA using the Juroshek method [3]. The complex-valued Γ_{gen}, Γ_{dut}, and Γ_{std} are used to correct for mismatch errors.

B.1.2 Uncertainty Budget

Table 6: Example uncertainty budget SP.

Source of relative uncertainty contribution	**Probab. distrib.**	**Relative standard uncertainty at frequency [GHz]**					
		0.05	0.1	2	26.5	30	40
Calibration factor of standard sensor	Normal	0.00253	0.00254	0.00311	0.01096	0.01215	0.01007
Drift of standard sensor	Normal	0.00076	0.00076	0.00093	0.00329	0.00365	0.00302
Mismatch	Normal	0.00017	0.00015	0.00017	0.00318	0.00357	0.00276
Monitor linearity, DUT connected	Normal	0.00018	0.00018	0.00018	0.00018	0.00018	0.00021
Monitor linearity, standard connected	Normal	0.00018	0.00018	0.00018	0.00018	0.00018	0.00021
DUT resolution	Uniform	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007
Monitor resolution, DUT connected	Uniform	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007
Monitor resolution, standard connected	Uniform	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007
Standard resolution	Uniform	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007
Root sum square of type B uncert.	Normal	0.00266	0.00267	0.00326	0.01188	0.01319	0.01087
Transfer of uncertainties from the normalising frequency		0	0.00266	0.00266	0.00266	0.00266	0.00266
Random errors	Normal	0	0.00008	0.00034	0.00020	0.00032	0.00020
Combined relative standard uncert.	Normal	0	0.0038	0.0042	0.0122	0.0135	0.0112
Expanded relative uncertainty	Normal	0	0.0075	0.0085	0.0244	0.0269	0.0224
Expanded uncertainty	Normal	0	**0.0075**	**0.0083**	**0.023**	**0.025**	**0.020**
Calibration factor		1	**0.9982**	**0.9770**	**0.931**	**0.924**	**0.914**

The example uncertainty budget corresponds to the data SP2 according to Tables 2 and 11. All uncertainty contributions above are relative. Therefore, all sensitivity coefficients are unity and are left out.

B.2 PTB

B.2.1 Method of calibration

Fig. 13 shows the direct comparison measurement setup of PTB. Three thermistor standards (covering the frequency ranges 50 MHz to 18 GHz, 18 GHz to 26.5 GHz, and 26.5 GHz to 40 GHz) and the DUT are alternately connected to one of the output ports of a power splitter. The DUT/standard power measurement gives the readouts $P_{ind,X}$ and $P_{ind,N}$, respectively. Simultaneously, the incident power level is monitored by a reference power sensor (indicated power $P_{ind,G}$) via the second output port of the power splitter.

The calibration factor $\eta_{cal,N}$ of the thermistor standards was determined at PTB by long-term microcalorimeter measurements resulting in low uncertainties. While the standard for the frequency range 50 MHz to 18 GHz has an N-male connector, the two other thermistors are equipped with waveguide flanges.

To mate the standards with the power splitter reference plane (2.92 mm connector), coax-to-coax and waveguide-to-coax adapters, respectively, are used. The S-parameters of the adapters are measured separately using a VNA. From the calibration factor $\eta_{cal,N}$ of the thermistors, their complex input reflection coefficient, and the adapter S-parameters,

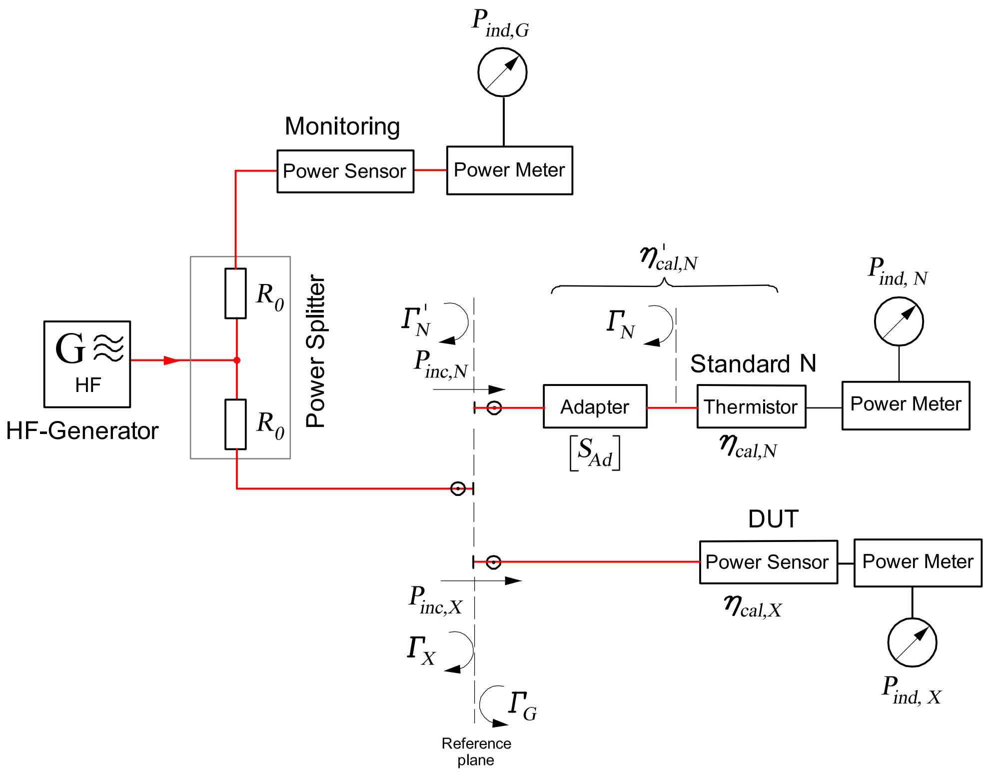

Fig 13: Direct comparison setup applied at PTB.

the calibration factor $\eta'_{cal,N}$ and input reflection coefficient Γ'_N of the combined adapter-thermistor standard are calulated. A cross-check between the calulated input reflection coefficient Γ'_N of the combined standard and its directly measured value gives an indication of the validity of the adapter measurement.

The equivalent source reflection coefficient Γ_G of the power splitter is determined by applying the Juroshek-Method [3] followed by a sophisticated second-order calibration using a high precision airline [4], [5]. Only for the second measurement PTB2 applied this second order calibration procedure in conjunction with a slightly different adapter characterisation method.

Knowing the complex-valued source reflection coefficient and the DUT/standard input reflection coefficient, a correction of the mismatch error is performed.

Since the overall frequency range was subdivided into the three subsets, at the turnover frequencies 18 GHz and 26.5 GHz the calibration factor was obtained using the different thermistor standards for the adjacent frequency ranges. A maximum difference of 0.5% indicates consistency of the thermistor standard calorimeter calibration.

B.2.2 Uncertainty Budgets

Table 7: Example uncertainty budget at 2 GHz.

$f = 2\,\mathrm{GHz}$					
Source of uncertainty	**Qty.**	**Probab. distrib.**	**Std. uncert.**	**Sensitivity coeff.**	**Uncert. contrib.**
Calibration factor of standard sensor	$\eta'_{cal,N}$	normal	0.00204	1.0005	0.00204
Drift of standard sensor	$\Delta\eta'_{cal,N}$	uniform	0.00059	1.0005	0.00059
Calibration factor of standard sensor @f_{ref}	$\eta'_{cal,N}(f_{ref})$	normal	0.00198	-0.9765	0.00194
Drift of standard sensor @f_{ref}	$\Delta\eta'_{cal,N}(f_{ref})$	uniform	0.00057	-0.9765	0.00056
Mismatch factor	M_{GX}/M_{GN}	U-shaped	0.00033	0.9703	0.00032
Mismatch factor @f_{ref}	M_{GX}/M_{GN}	U-shaped	0.0002	0.9700	0.0002
Power ratio DUT	$\frac{P_{ind,X}}{P_{ind,G,X}}$	normal	0.0054	0.9734	0.00052
Power ration Standard	$\frac{P_{ind,N}}{P_{ind,G,N}}$	normal	0.0002	-0.9724	0.00019
Power ratio DUT : Nonlinearity, Drift, Resolution of DUT PM[a]		uniform	0.0003	0.9745	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of Std. PM		uniform	0.0005	-0.9745	0.00049
Power ratio DUT: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	-0.9745	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	0.9745	0.00029
Combined standard uncert.					0.00305
Expanded uncertainty					**0.0061**
Calibration factor	$\boldsymbol{\eta_{cal,X,50\mathrm{MHz}}}$				**0.9761**

[a]Power Meter

Table 8: Example uncertainty budget at 10 GHz.

$f = 10\,\mathrm{GHz}$					
Source of uncertainty	**Qty.**	**Probab. distrib.**	**Std. uncert.**	**Sensitivity coeff.**	**Uncert. contrib.**
Calibration factor of standard sensor	$\eta'_{cal,N}$	normal	0.00244	1.0119	0.00247
Drift of standard sensor	$\Delta\eta'_{cal,N}$	uniform	0.00070	1.0119	0.00071
Calibration factor of standard sensor @f_{ref}	$\eta'_{cal,N}(f_{ref})$	normal	0.00198	-0.9638	0.00191
Drift of standard sensor @f_{ref}	$\Delta\eta'_{cal,N}(f_{ref})$	uniform	0.00057	-0.9638	0.00055
Mismatch factor	M_{GX}/M_{GN}	U-shaped	0.00065	0.9579	0.00062
Mismatch factor @f_{ref}	M_{GX}/M_{GN}	U-shaped	0.00020	0.9574	0.00019
Power ratio DUT	$\frac{P_{ind,X}}{P_{ind,G,X}}$	normal	0.00016	0.9516	0.00015
Power ration Standard	$\frac{P_{ind,N}}{P_{ind,G,N}}$	normal	0.00005	-0.9617	0.00005
Power ratio DUT : Nonlinearity, Drift, Resolution of DUT PM[a]		uniform	0.0003	0.9620	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of Std. PM		uniform	0.0005	-0.9620	0.00048
Power ratio DUT: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	-0.9620	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	0.9620	0.00029
Combined standard uncert.					0.00335
Expanded uncertainty					**0.0067**
Calibration factor	$\boldsymbol{\eta_{cal,X,50\mathrm{MHz}}}$				**0.9524**

Table 9: Example uncertainty budget at 20 GHz.

$f = 20\,\mathrm{GHz}$					
Source of uncertainty	**Qty.**	**Probab. distrib.**	**Std. uncert.**	**Sensitivity coeff.**	**Uncert. contrib.**
Calibration factor of standard sensor	$\eta'_{cal,N}$	normal	0.00401	1.0340	0.00415
Drift of standard sensor	$\Delta\eta'_{cal,N}$	uniform	0.00116	1.0340	0.00120
Calibration factor of standard sensor @f_{ref}	$\eta'_{cal,N}(f_{ref})$	normal	0.00198	-0.9728	0.00193
Drift of standard sensor @f_{ref}	$\Delta\eta'_{cal,N}(f_{ref})$	uniform	0.00057	-0.9728	0.00056
Mismatch factor	M_{GX}/M_{GN}	U-shaped	0.00211	0.9863	0.00208
Mismatch factor @f_{ref}	M_{GX}/M_{GN}	U-shaped	0.0002	0.9663	0.00019
Power ratio DUT	$\frac{P_{ind,X}}{P_{ind,G,X}}$	normal	0.00021	0.9509	0.00020
Power ration Standard	$\frac{P_{ind,N}}{P_{ind,G,N}}$	normal	0.00019	-1.0016	0.00019
Power ratio DUT : Nonlinearity, Drift, Resolution of DUT PM[a]		uniform	0.0003	0.9710	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of Std. PM		uniform	0.0005	-0.9710	0.00049
Power ratio DUT: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	-0.9710	0.00029
Power ratio Standard: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	0.97010	0.00029
Combined standard uncert.					0.00552
Expanded uncertainty					**0.0105**
Calibration factor	$\boldsymbol{\eta_{cal,X,50\mathrm{MHz}}}$				**0.9408**

Table 10: Example uncertainty budget at 40 GHz.

$\boldsymbol{f = 40\,\mathrm{GHz}}$					
Source of uncertainty	**Qty.**	**Probab. distrib.**	**Std. uncert.**	**Sensitivity coeff.**	**Uncert. contrib.**
Calibration factor of standard sensor	$\eta'_{cal,N}$	normal	0.00351	1.0227	0.00359
Drift of standard sensor	$\Delta\eta'_{cal,N}$	uniform	0.00101	1.0227	0.00104
Calibration factor of standard sensor @f_{ref}	$\eta'_{cal,N}(f_{ref})$	normal	0.00198	-0.9318	0.00185
Drift of standard sensor @f_{ref}	$\Delta\eta'_{cal,N}(f_{ref})$	uniform	0.00057	-0.9318	0.00053
Mismatch factor	M_{GX}/M_{GN}	U-shaped	0.00454	0.9617	0.00436
Mismatch factor @f_{ref}	M_{GX}/M_{GN}	U-shaped	0.0002	0.9256	0.00019
Power ratio DUT	$\frac{P_{ind,X}}{P_{ind,G,X}}$	normal	0.00182	0.9067	0.00165
Power ration Standard	$\frac{P_{ind,N}}{P_{ind,G,N}}$	normal	0.00116	-0.9617	0.00112
Power ratio DUT : Nonlinearity, Drift, Resolution of DUT PM[a]		uniform	0.0003	0.9301	0.00028
Power ratio Standard: Nonlinearity, Drift, Resolution of Std. PM		uniform	0.0005	-0.9301	0.00047
Power ratio DUT: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	-0.9301	0.00028
Power ratio Standard: Nonlinearity, Drift, Resolution of monitor PM		uniform	0.0003	0.9301	0.00028
Combined standard uncert.					0.0064
Expanded uncertainty					**0.0128**
Calibration factor	$\boldsymbol{\eta_{cal,X,50\mathrm{MHz}}}$				**0.9111**

C Cross Check Measurement of the Travelling Standard

To investigate the stability of the travelling standard during the whole measurement period a cross check was made between the measurements SP1 and SP2 according to Table 2. The compared data and the result is shown in Table 11 and in Figs. 14 to 16.

The limit for the allowable drift in the travelling standard was calculated from the uncertainty contributions to $u(\eta_{\text{SP1}})$ and $u(\eta_{\text{SP2}})$. However, no systematic contributions were included. E.g. the uncertainty in the calibration factor of the standard sensor was not included. The reduced uncertainties are denoted $u'(\eta_{\text{SP1}})$ and $u'(\eta_{\text{SP2}})$. In $u'(\eta_{\text{SP1}})$ uncertainty contributions due to resolution, repeatability, and linearity were included. In $u'(\eta_{\text{SP2}})$ uncertainty contributions due to drift, resolution, repeatability, and linearity were included.

Since $E_n < 1$ the cross check is successful. Nevertheless, there is a difference in the calibration factor of the DUT between the two measurements. Redundant data from the measurement setup at SP shows that the setup and the standard sensor have been stable throughout the comparison. Thus, the deviation is caused by drift in the calibration factor of the DUT. To accomodate for this drift when comparing the SP and PTB data the average of the SP1 and SP2 data was taken and compared to the PTB1 and PTB2 data.

Table 11: Measurement data SP1 - SP2.

Frequency GHz	E_n	Error	U(Error) k=2	η_{SP1}	$u(\eta_{SP1})$	$u'(\eta_{SP1})$ [b]	η_{SP2}	$u(\eta_{SP2})$	$u'(\eta_{SP2})$ [c]
0.05	0	0	0	1	0	0	1	0	0
0.10	0.01	0.0000	0.0025	0.9982	0.0038	0.0004	0.9982	0.0038	0.0012
0.50	0.01	0.0000	0.0027	0.9926	0.0041	0.0004	0.9926	0.0041	0.0013
1.00	0.00	0.0000	0.0027	0.9864	0.0041	0.0004	0.9864	0.0041	0.0013
2.00	0.16	0.0004	0.0028	0.9765	0.0041	0.0005	0.9770	0.0041	0.0013
3.00	0.52	0.0015	0.0029	0.9705	0.0041	0.0007	0.9720	0.0041	0.0013
4.00	0.46	0.0013	0.0029	0.9653	0.0041	0.0006	0.9666	0.0041	0.0013
5.00	0.27	0.0007	0.0027	0.9625	0.0041	0.0004	0.9633	0.0041	0.0013
6.00	0.30	0.0009	0.0030	0.9584	0.0045	0.0004	0.9593	0.0045	0.0014
7.00	0.33	0.0010	0.0030	0.9558	0.0045	0.0004	0.9568	0.0045	0.0014
8.00	0.45	0.0014	0.0030	0.9543	0.0045	0.0004	0.9556	0.0045	0.0014
9.00	0.54	0.0016	0.0030	0.9536	0.0045	0.0004	0.9552	0.0045	0.0014
10.00	0.56	0.0018	0.0032	0.9492	0.0050	0.0004	0.9510	0.0050	0.0016
11.00	0.65	0.0021	0.0032	0.9477	0.0049	0.0004	0.9497	0.0050	0.0016
12.00	0.72	0.0023	0.0032	0.9475	0.0050	0.0004	0.9498	0.0050	0.0016
13.00	0.74	0.0026	0.0035	0.9429	0.0054	0.0004	0.9454	0.0054	0.0017
14.00	0.65	0.0024	0.0038	0.9417	0.0059	0.0004	0.9441	0.0059	0.0018
15.00	0.73	0.0026	0.0035	0.9415	0.0054	0.0004	0.9441	0.0054	0.0017
16.00	0.76	0.0027	0.0035	0.9384	0.0054	0.0004	0.9411	0.0054	0.0017
17.00	0.76	0.0029	0.0038	0.9366	0.0059	0.0004	0.9395	0.0059	0.0019
18.00	0.77	0.0029	0.0038	0.9379	0.0059	0.0004	0.9408	0.0059	0.0019
19.00	0.59	0.0029	0.0050	0.9401	0.0079	0.0004	0.9430	0.0079	0.0024
20.00	0.56	0.0028	0.0050	0.9387	0.0079	0.0004	0.9415	0.0079	0.0024
21.00	0.54	0.0027	0.0050	0.9354	0.0079	0.0004	0.9381	0.0079	0.0024
22.00	0.54	0.0028	0.0053	0.9345	0.0084	0.0004	0.9373	0.0084	0.0026
23.00	0.50	0.0028	0.0056	0.9351	0.0089	0.0004	0.9379	0.0090	0.0027
24.00	0.46	0.0027	0.0059	0.9318	0.0095	0.0004	0.9345	0.0096	0.0029
25.00	0.46	0.0029	0.0062	0.9317	0.0101	0.0004	0.9346	0.0102	0.0031
26.00	0.43	0.0028	0.0066	0.9297	0.0107	0.0005	0.9325	0.0108	0.0032
26.50	0.44	0.0031	0.0069	0.9283	0.0112	0.0004	0.9314	0.0113	0.0034
27.00	0.41	0.0029	0.0072	0.9255	0.0118	0.0004	0.9285	0.0119	0.0036
28.00	0.46	0.0035	0.0075	0.9240	0.0123	0.0004	0.9275	0.0124	0.0037
29.00	0.52	0.0039	0.0075	0.9247	0.0123	0.0005	0.9287	0.0125	0.0037
30.00	0.49	0.0037	0.0076	0.9207	0.0123	0.0005	0.9244	0.0124	0.0038
31.00	0.40	0.0030	0.0076	0.9215	0.0123	0.0004	0.9245	0.0124	0.0038
32.00	0.37	0.0027	0.0072	0.9269	0.0117	0.0004	0.9296	0.0118	0.0036
33.00	0.45	0.0031	0.0069	0.9279	0.0112	0.0004	0.9310	0.0113	0.0034
34.00	0.52	0.0037	0.0072	0.9216	0.0117	0.0004	0.9253	0.0117	0.0036
35.00	0.50	0.0035	0.0069	0.9142	0.0111	0.0004	0.9177	0.0112	0.0034
36.00	0.35	0.0024	0.0069	0.9143	0.0111	0.0005	0.9167	0.0112	0.0034
37.00	0.38	0.0026	0.0069	0.9214	0.0112	0.0004	0.9240	0.0113	0.0034
38.00	0.40	0.0026	0.0066	0.9225	0.0108	0.0005	0.9251	0.0108	0.0033
39.00	0.43	0.0029	0.0066	0.9200	0.0107	0.0004	0.9229	0.0108	0.0033
40.00	0.50	0.0032	0.0064	0.9110	0.0102	0.0005	0.9142	0.0102	0.0032

[b] Only uncertainties due to resolution, repeatability, and linearity are included.

[c] Only uncertainties due to drift, resolution, repeatability, and linearity are included.

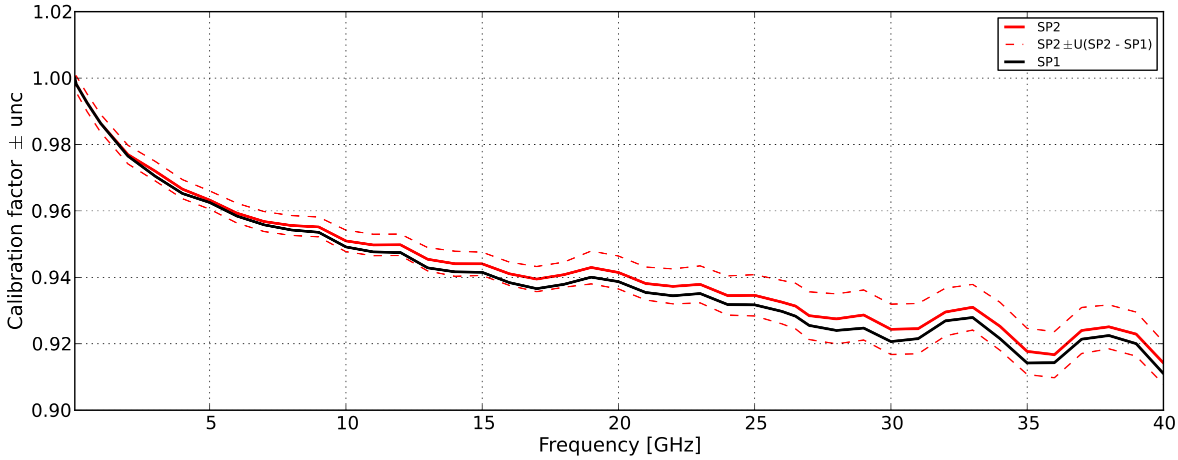

Fig 14: Calibration factors η_{SP1} and η_{SP2} and their expanded uncertainties.

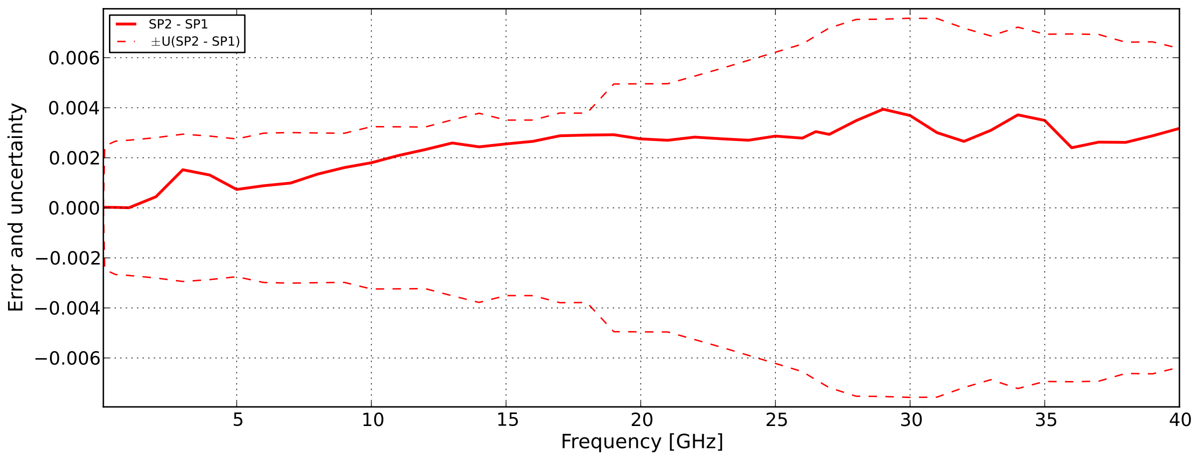

Fig 15: Error between η_{SP1} and η_{SP2} and the combined expanded uncertainty.

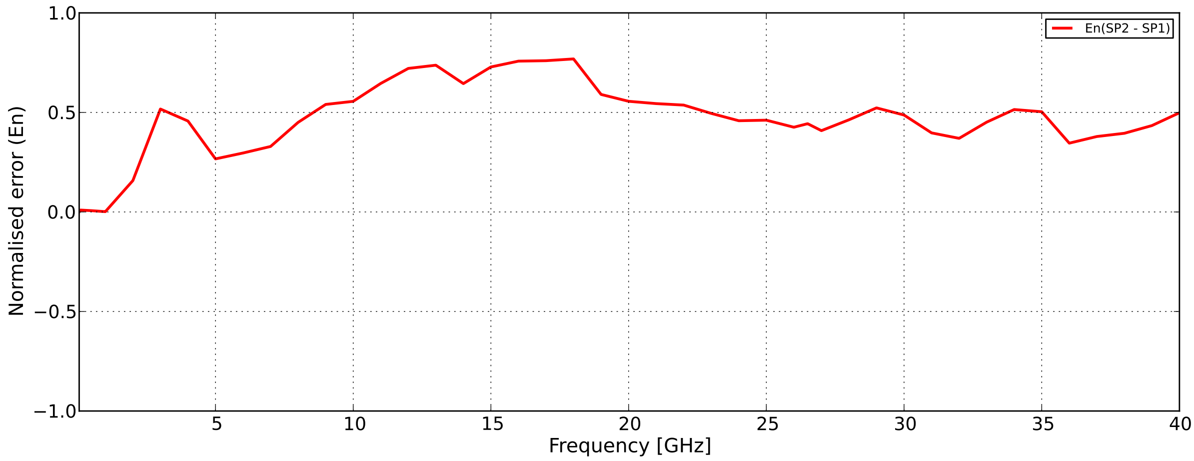

Fig 16: Normalised error between η_{SP1} and η_{SP2}.

References

[1] J. Juroshek, "0.05 - 50 GHz Direct Comparison Power Calibration System," Conference on Precision Electromagnetic Measurements, Sydney, Australia, May 2000.

[2] "Guide to the expression of uncertainty in Measurement (GUM)", *International Organisation of Standardisation*, Geneva, 1995.

[3] J. Juroshek, "A direct calibration method for measuring equivalent source match," *Microwave Journal*, pp. 106-118, Oct. 1997.

[4] G. Wübbeler, C. Elster, T. Reichel, R. Judaschke, "Determination of Complex Residual Error Parameters of a Calibrated Vector Network Analyzer," 69^{th} ARFTG Conference, Honolulu, USA, June 2007.

[5] G. Wübbeler, C. Elster, T. Reichel, R. Judaschke, "Determination of the Complex Residual Error Parameters of a Calibrated One-Port Vector Network Analyzer," *IEEE Trans. Instrum. Meas.*, vol. 58, no. 9, pp. 3238-3244, 2009.