## Report on

## EUROMET Project 371

## Angle calibration on precision polygons

## April 1996 - March 1999

Draft A

Physikalisch-Technische Bundesanstalt
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R. Probst, R. Wittekopf

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## 1. Introduction

Precision polygons are basic standards for angle measurement, which are used and calibrated by the national standards laboratories in particular. The uncertainty of measurement attainable with polygons largely depends on the geometry (flatness and squareness) of their reflecting faces and also on optical errors of the autocollimator and alignment errors occurring during sensing of the faces. A CCDM comparison of two 12-sided polygons, carried out by eleven laboratories between 1980 and 1986, produced unsatisfactory results, as the measurement differences turned out to be much greater than the uncertainties estimated by the participants [1].

In 1995, OFMET and PTB proposed a EUROMET project "Angle calibration on precision polygons,. It was decided that PTB should act as the pilot laboratory and that two polygons should be compared, one with 24 faces and the other with seven faces. Participation in this intercomparison was announced by nine European national institutes, five of which took part, together with PTB, in the first circulation between April 1996 and May 1997. A second circulation was organized between October 1997 and January 1999, with six other laboratories participating, two of which had asked at a later date to join this comparison.

It was the main objective of the project to provide greater clearness about the expression of uncertainty in angle calibrations on polygons, in accordance with the Guide [2]. Interferometric measurements of the polygon faces carried out at PTB were expected to provide information about the influence of the flatness quality on the uncertainty, on the basis of an analysis of the correlation with the laboratories' results. The interferometric method can finally be tested with a view to obtaining reference values for the true angles between the polygon faces, mathematically defined by the normal directions of the best-fitted planes derived from the interferograms of the faces.

This draft report covers the evaluation of the measurement results of the two circulations so far made available by eleven laboratories including PTB, together with summaries and extracts from the laboratories' reports in accodance with the measurement instructions for this project.

## 2. Participating laboratories and time schedule

The following table lists the participating national standards laboratories with their responsible metrologists and the periods during which measurements took place:

## Pilot laboratory:

PTB $\quad$| Physikalisch-Technische Bundesanstalt |
| :--- |
|  |
|  |
|  |
|  |
| Dundesallee 100 |
| D -38116 Braunschweig, Germany |

R. Probst January - March
R. Wittekopf
1996
May - August 1997
January - March 1999

Laboratories of the first circulation:
OFMET Swiss Federal Office of Metrology Lindenweg 50
R. Thalmann April - May 1996 CH-3003 Bern-Wabern, Switzerland

NMi Van Swinden Laboratory
Netherlands Measurements Institute Schoemakerstraat 97
NL-2628 VK Delft, Netherlands
VTT VTT Manufacturing Technology PL 1702
H. Lehto July - October 1996 FIN-02044 VTT, Finland

IMGC Istituto di Metrologia
"G. Colonnetti"
H. Haitjema

May - June
G. Kotte

1996

Strada delle Cacce, 73
I-10135 Torino, Italy
NPL National Physical Laboratory
G.N. Peggs March - May

Queens Road
D.R. Flack 1997

Teddington, Middlesex TW11 OLW
United Kingdom
Laboratories of the second circulation:
SMÚ Slowensky Metrologicky Ústav
Slovak Institute of Metrology
J. Mokros

October -
November 1997

## Karloveska 63

SK-84255 Bratislava, Slovakia
CEM Centro Español de Metrología
E. Prieto December 1997c/del Alfar, 2
E-28760 Tres Cantos
Madrid, Spain

| LNE | Laboratoire National d'Essais Bureau National de Métrologie <br> 1, Rue Gaston Boissier F-75724 Paris Cedex 15, France | M. Priel G.P. Vailleau | $\begin{aligned} & \text { March - May } \\ & 1998 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| IGM | Inspection Générale de la Métrologie, Metrology Service <br> Emile Jacqmainlaan 154 <br> B-1000 Brussels, Belgium | H. Pirée | May - August 1998, continued later |
| GUM | Glówny Urzad Miar Central Office of Measures <br> 2, Elektoralna Street PL-00-950 Warsaw, Poland | Z. Ramotowski <br> G. Rudnicka | August October 1998 |
| UME | Ulusal Metroloji Enstitüsü National Metrology Institute P.O. Box 21 TR-41470 Gebze-Kocaeli, Turkey | T. Yandayan | October 1998 January 1999 |
| CMI | Czech Metrological Institute OI Liberec, Slunecna 23, CZ - 46001 Liberec, Czech Republic | V. Stezka | July - <br> September 1999 |

IGM could not finish the measurements because of the delays in the time schedule, and it has been unable to date to report any results. IGM has meanwhile continued the measurements; the results will be added to this report later.

In November 1998, another application was filed with PTB by the Czech Metrology Institute (CMI), asking to join this comparison. An exception was made and CMI admitted as the last participant, mainly for political reasons, because the Slovak Institute SMÚ took also part. The CMI measurements will be performed after those of IGM, and the results will also be added to the final version of this report.

## 3. Precision polygons as transfer standards

The following polygons were used as transfer standards for the comparison: A 7sided polygon (provided by NMi) and a 24 -sided polygon (provided by PTB); their main features are listed in the following table:

|  | 7-sided polygon (P7) | 24-sided polygon (P24) |
| :---: | :---: | :---: |
| Manufacturer, identification | Rank Taylor Hobson Ltd. SP LE 5997 | Möller / Wedel PTB 5.22-23-539-1 |
| Pitch angle | $51^{\circ} 25^{\prime} 42,857{ }^{\prime \prime} .$. | $15^{\circ}$ |
| Size of reflecting faces | $15 \mathrm{~mm} \varnothing$ | $20 \mathrm{~mm} \times 25 \mathrm{~mm}$ (width $x$ height) |
| Diameter | 60 mm | 150 mm |
| Material | glass | glass ceramics "Zerodur" |
| Mass (incl. case) | 495 g | 2126 g |
| Angle deviations from nominal (max.-min.) | 2,0" | 2,0" |
| Pyramidality of faces ${ }^{1)}$ (max.-min.) | 8,4" | 8,2" |
| Mean flatness of faces ${ }^{1)}$ | $\begin{aligned} & 9 \mathrm{~nm} \text { P-V } \\ & 2 \mathrm{~nm} \text { RMS } \end{aligned}$ |  |

P7 was made to comply with NPL specification MOY / SCMI / 83 Issue 5 dated October 1969. Both polygons have central mounting holes and are accommodated in metal cases. An adjustable mounting device, which was circulated together with the polygons, could be used to facilitate the alignment of the polygons in the measurement set-up (see Annex A). The case of P7 had circular openings which limited the diameter of the reflecting faces to 15 mm , whereas the case of P24 had an open perimeter exposing the faces to their full rectangular size of $20 \mathrm{~mm} x$ 25 mm . The rather high P-V flatness deviations of these faces are due to roll-off close to the edges, which is significantly reduced when the faces are masked to 19 mm diameter (see Section 6.5).

The faces of both polygons are numbered on their cases, starting with No.1. The polygons can also be mounted in the reversed position (numbers on the bottom side), the counting direction with respect to the rotation of the table then being reversed. The stability of the polygon standards was checked by three repeated calibrations performed by the pilot laboratory in the course of the intercomparison.

These calibrations were found identical within the uncertainty of measurement, so the stability of the standards has been verified.

## 4. Description of the task

The aims of the comparison were the following:

- to determine the mutual compatibility of angle calibrations on precision polygons among laboratories realizing angular units independently and applying different methods of measurement;
- to improve the expression of uncertainty in angle measurements of this kind according to the "Guide" [ 2 ];
- to investigate the potential existence of systematic errors and to determine the limit of accuracy attainable with polygon / autocollimator systems;
- to exchange information about measurement techniques and procedures.

The results of this EUROMET comparison must also be considered with regard to the planned "CCL Key Comparison" of angle standards, which will be organized by CSIR, South Africa, and will take place soon.

The laboratories applying for participation in this comparison first gave a short description of the envisaged extent of the measurements, the available instrumentation and the methods they intend to use. Recommendations were made by the pilot laboratory as regards the measurement instructions which were distributed in advance to all participants (see Annex A). Several alternative measurements were suggested for the comparison, taking into account the different capabilities of the laboratories:

- Measurement of both polygons or of only one of them
- Measurement in all faces of P24 or restriction to 12 faces
- Measurement of the polygons in normal position or in inverted position as well
- Measurement by applying standard methods 1 and 2 or other methods
- Use of the participant's own mounting device instead of the device made available.

The request had been to report the results of the measurements together with the combined standard uncertainty on special report forms, and to add the description of
the adjustment conditions, measuring instruments and measurement procedures and the evaluation of uncertainties. This requirement was met by the laboratories, more detailed supplementary reports being given as well.

## 5. Measurement methods, conditions and equipment

Table A gives a summary of the methods and conditions of measurement and states the expanded uncertainties in accordance with the laboratories' reports. Letters indicate which of polygons P7 and P24 was measured exclusively in the normal (n) or in the inverted (i) position; where no letter is indicated, the measurements were performed in both positions. The indication P24/2 means that P24 was measured on 12 faces only.

Method 1 refers to the use of a single autocollimator, either by the cross calibration method or by the method of direct comparison with an angle measuring or indexing table. In method 2, use is made of two autocollimators and a rotary table for angular positioning.

Table B gives a survey of the measuring instruments of the laboratories. NMi did not use the measuring system RON 905 for the measurement but only for positioning of the polygons (method 2). NPL used two different Moore indexing tables, one with 1440 serrations for P24 and the other with 2160 serrations for P7, together with two different autocollimator types. Two additional autocollimators of the same type were set up with their measuring axis vertically oriented to control the levelling of the polygons.

In addition to tables $A$ and $B$, the summary below gives the calibration procedures followed by the laboratories.

The cross calibration method is often used in such a way that changes of the relative positions (so-called settings) of the polygon in relation to the angle measuring or indexing table are allowed by an additional indexing table. This additional table can be used either as an "intermediate" (between polygon and measuring table) or as a "subsidiary" (beneath the measuring table supporting the polygon).

NPL reported that the polygon was levelled again at the start of each set of measurements after a setting had been made, to minimize the pyramid error. The other laboratories did not report anything in this respect; it is assumed that the pyramid errors stated in the reports were maintained for all polygon settings.

## PTB:

P24, P7:
(1) Cross calibration vs. numerically controlled rotary table with measuring system RON 905, polygon settings with intermediate Moore 1440 (P24) or manually on mounting device (P7)
(2) Cross calibration vs. new angle comparator WMT 220, polygon setting with numerically controlled calibration facility of the WMT 220.

## OFMET:

P24, P7 n:
Cross calibration vs. numerically controlled rotary table / RON 905, polygon settings with intermediate Moore 1440 (P24) or manually on mounting device (P7n)

P7 i:
Direct comparison vs. rotary table / RON 905

## NMi:

P24, P7:
NMi version of method 2: Comparison of polygon vs. two autocollimators
(a) directed at two neighbouring faces
(b) directed at two faces closest to $180^{\circ}$

In both cases measurements are performed with the two autocollimators while the polygon is rotated on the rotary table SIP MU-214B step by step over $360^{\circ}$, repeated in three rounds.

## VTT:

P24 n:
Cross calibration vs. rotary table / RON 905, setting procedure not described.

P7 n:
Direct comparison vs. rotary table / RON 905 whose errors were corrected with results of cross calibration P24n, three times in one position.

## IMGC:

P24 n, P24/2 i:
Cross calibration vs. Moore 1440, repeated in reversed order of rotation, polygon settings with subsidiary indexing table.

P7:
Direct comparison of each pair of adjacent polygon faces vs. the same angular interval of a Moore 1440, repeated at $0^{\circ}, 90^{\circ}, 180^{\circ}$ and $270^{\circ}$ reading of the indexing table. Measurement set up with subsidiary rotating table (SIP) and auxiliary Moore table.

Circular aperture diaphragms 26 mm in diameter for P24 and 16 mm in diameter for P7 were used in front of the autocollimators. The faces of P24 thus were slightly cut off at the corners.

## NPL:

P24/2:
Cross calibration vs. Moore 1440, repeated for two revolutions at each setting (manual settings on mounting device). At each setting the polygon was levelled.

## P7:

Direct comparison vs. Moore 2160 with step-by-step index settings either $51^{\circ} 30^{\prime}$ or $51^{\circ} 20^{\prime}$, difference readings with autocollimator $\mathrm{MO}(1) 1873$, repeated for two readings at each setting, repeated in $0^{\circ}, 120^{\circ}$ and $240^{\circ}$ start reading of the indexing table. At each $120^{\circ}$ setting the polygon was levelled.

## SMÚ:

P24, P7
Cross calibration vs. ring laser by use of goniometer GS1L in dynamic mode of operation (continuous rotation), 5 revolutions in both directions of rotation, repeated 15 times. Additional measurement of pyramidality, adjustment on mounting device of the goniometer table.

## CEM:

P24/2, P7

1. Cross calibration vs. measuring table ("angular generator") and use of autocollimator DA 80
2. Two-autocollimator method by use of DA 80 and DA 400 in all $n$ settings ( $n=12$, $\mathrm{n}=7$ ), readings taken from the autocollimators after stepwise positioning of the table.

## LNE:

P24 n
Cross calibration vs. angle measuring table in 24 positions of the polygon, measured ten times in each position, each time clockwise (CW) and counter-clockwise (CCW), autocollimator used in $\pm 10$ " range.

## P 24/2 i

Direct comparison with the angle measuring table after applying the table corrections obtained by previous method P24 n; four measurements with one unchanged setting of the polygon on the table, each measurement CW and CCW.

## P7

Measured by using the correction values derived from the previous measurement of P24. Ten repeated measurements (CW and CCW) with one polygon setting.

## GUM:

P24, P7
Two-autocollimator method using $2 \times$ FET 100 and table TL 78 for positioning. n/2 settings of the instruments relative to the $\mathrm{n}=24$ and $\mathrm{n}=7$ faces, pyramidality adjustment by use of a DA 80 autocollimator and GUM's own polygon mounting device. An aperture diaphragm 15 mm in diameter was used with the autocollimators, thus limiting the faces of P24.

## UME:

P24/2
Cross calibration vs. Moore 1440 by use of ELCOMAT HR ( $12 \times 12$ measurements), polygon settings by use of another Moore 1440 as a subsidiary indexing table

## P7

Method proposed by H. Haitjema, as described in Annex B (according to UME report).

## 6. Results and discussion

### 6.1 General remarks

The reported cumulative angle deviations $\Delta \beta_{i}$ relate to the datum face marked 1. Any bias error in the measurement of this face thus enters as a constant into all the values of $\Delta \beta_{i}$, which can lead to constant differences between the laboratories' results (an example of this is shown in Section 6.2, Figure 1.1). To eliminate this dependence on the error of a single face, it is therefore preferred to relate the values of $\Delta \beta_{i}$ to the average of all faces. These values are called reduced angle deviations $\Delta \beta_{i}{ }^{r}$ :

$$
\begin{equation*}
\overline{\Delta \beta}_{i}=\Delta \beta_{i}-\frac{1}{n} \sum_{k=1}^{n} \Delta \beta_{k}, \quad i=1 \ldots \mathrm{n}: \text { Face No. } \tag{1}
\end{equation*}
$$

From this definition it follows that $\sum_{i=l}^{n} \Delta \beta_{i}^{r}=0$. The angle deviation between any two faces i and j is equal to the difference between their reduced angle deviations $\Delta \beta_{i}{ }^{r}$ and $\Delta \beta_{j}{ }^{r}$. An advantage of the reduced angle deviation must be seen in the fact that this value relates to a single polygon face, contrary to the pitch or cumulative angle values which always relate to a pair of faces. The reduced angle deviation can thus be correlated with the quality and error influence of the specific face.

After the laboratories' results $\Delta \beta_{i}$, in the first step of the evaluation, the results $\Delta \beta_{i}{ }^{r}$ are given as absolute values. In the following, the differences of the values $\Delta \beta_{i}{ }^{r}$ from reference values are given, which are calculated as the arithmetic mean of all the results. These differences are reported together with the uncertainties stated by the laboratories.

The described evaluation was made only on the basis of the results obtained with the polygons in the normal position (marking on top). In addition to this, the differences between the results of the normal and inverted positions, as far as reported by the laboratories, have been evaluated and compared separately. These differences are also calculated from reduced angle deviations reported by each participant, but not related to common mean values. In the following sections 6.2 and 6.3 , the results for the two polygons in their normal positions are presented, in section 6.4 the differences between their normal and inverted positions.

Finally, in Section 6.5, the interferometric measurements of flatness and pyramidality of the polygon faces are presented and these results compared in Section 6.6 with the angle results using a correlation analysis. The last Section 6.7 reports about investigations on autocollimator influences revealed by this intercomparison.

### 6.2 24-sided polygon

In Table 1 the laboratories' absolute results of the cumulative angle deviations $\Delta \beta_{i}$, related to face 1, are listed for the polygon in its normal position. Four of the laboratories restricted their measurements to only 12 polygon faces, these results being comparable with the corresponding results of the other laboratories. In the last row of the table the expanded uncertainties $U_{j}(k=2)$ as reported by the laboratories are given. The last two columns contain the arithmetic mean of the $\Delta \beta_{i}$ and the standard deviation $\mathrm{s}_{\mathrm{i}}$, calculated from all results for each face i .

The diagrams of Figures 1.1 and 1.2 demonstrate the argument given in Section 6.1 for the introduction of the reduced angle deviations $\Delta \beta_{i}{ }^{r}$. Figure 1.1 shows the twelve results $\Delta \beta_{i}$ of Table 1 for the twelve odd-numbered faces. Three of these results have a distinct offset against the other nine, which is apparently caused by a predominant deviation of the result for face 1. In the corresponding reduced angle deviations $\Delta \beta_{i}{ }^{r}$ (Figure 1.2), this discrepancy is largely reduced, the results coming to a better coincidence.

Table 2 gives the differences of the reduced angle deviations $\Delta \beta_{i}{ }^{r}$ from their arithmetic mean over the twelve results according to Figure 1.2. In the last column of this scheme again the standard deviations $s_{i}$ of the results are listed for each face. In the last row the standard deviations $\mathrm{s}_{\mathrm{j}}$ calculated from all the faces are additionally given for each result.

The graphical representation of these results, is given in Figures 2.1 to 2.3, separately for each of the faces, together with the expanded uncertainties $U_{j}$ of the laboratories, indicated by double bars. Most remarkable in these diagrams are some larger and closely coincident deviations of the results of NMi, CEM1 and CEM2 from the majority of the other results, which, due to this, are all slightly shifted to the other side of the arithmetic mean. This gave rise to discussions between the laboratories
of NMi and CEM and the pilot laboratory and a decision about additional investigations of autocollimator influences, which are dealt with in Section 6.7.

For the eight laboratories measuring this polygon on all 24 faces, the differences of the results $\Delta \beta_{i}{ }^{r}$ from their arithmetic mean are listed in Table 3, again together with the standard deviations $\mathrm{s}_{\mathrm{i}}$ and $\mathrm{s}_{\mathrm{j}}$ according to Table 2. The corresponding graphical representations of these results are shown in Figures 3.1 to 3.6.

### 6.3 7-sided polygon

The absolute results of the cumulative angle deviations $\Delta \beta_{i}$, related to face 1 and measured in the normal position of this polygon, are given in Table 4, together with the reported expanded uncertainties $\mathrm{U}_{\mathrm{j}}(\mathrm{k}=2)$. Figure 4.1 shows a graphical representation of these results. A distinct offset especially of one result against all the others can be seen. This discrepancy is almost eliminated when the results are transformed to the reduced angle deviations $\Delta \beta_{i}^{r}$ (Figure 4.2).

The differences of the reduced angle deviations from the arithmetic mean, obtained from the twelve results for each face, are listed in Table 5. In addition, the standard deviations $s_{i}$ and $s_{j}$ as defined above are given.

Figures 5.1 and 5.2 show these results with their expanded uncertainties $U_{j}(k=2)$ in a graphical form, separately for each of the faces.

### 6.4 Differences between polygon positions

The differences between the polygon angles measured in the two positions marking up (normal position) and marking down (inverted position) - are summarized in the following tables for both polygons as far as reported by the laboratories.

In Table 6 twelve results are given for the 7 -sided polygon, together with the standard deviations $\mathrm{s}_{\mathrm{j}}$ for each result as well as the mean and standard deviations $\mathrm{s}_{\mathrm{i}}$ for each face.

In Table 7 eleven results are listed in the same way for the 24 -sided polygon measured on twelve faces and finally five results in Table 8 for the measurements on 24 faces.

The comparison of the values $s_{i}$ and $s_{j}$ in Tables 6, 7 and 8 with the corresponding values in Tables 2, 3 and 5 reveal a remarkable difference between the two polygons: In the case of P7 these values are of the same magnitude, whereas for P24 the values in Tables 2 and 3 are often much larger than those of Tables 7 and 8 . The differences between the polygon positions therefore do not help to evaluate the uncertainty of measurements as had been expected.

### 6.5 Interferometric measurements

In the pilot laboratory, measurements on the two polygons were made by the method described in [3], [4] using a phase shifting interferometer (PI). One aim of this method is to obtain the flatness deviations of the faces, shown in Annex C in the form of contour line and profile diagrams. For the 24 -sided polygon, the interferograms were evaluated for the full rectangular size of the faces as well as for a circular aperture limited to $18,6 \mathrm{~mm}$ diameter.

In the case of circular apertures, the Zernike polynomial evaluation of the phase map data enables to obtain also the pyramidal and reduced angle deviations of the polygons from the tilt angles of the first-order fitted planes.

Tables 9 and 10 give the resulting RMS and P-V values, the pyramidal deviations and the differences between the reduced angle deviations measured with the Elcomat HR and the PI, in the case of P24 for both sizes of the faces. With unlimited faces the flatness deviations of P24, expressed by the P-V values, are rather large due to roll-off close to the edges. These are greatly reduced when the circular aperture is applied. For these cases, Tables 9 and 10 give the RMS and P-V values as differences from their mean values, which show that the flatness quality of P7 is about twice better than that for P 24 .

The differences between the HR and PI results of the reduced angle deviations, given in the last two columns of Table 9, relate to HR measurements without and with an aperture limitation to about 18 mm and thus reveal the influence of aperture errors of the autocollimator. The comparison between these results shows that the aperture influence is rather undefined and does not generally reduce the differences to the PI results but can even increase them. The differences HR-PI are always within $\pm 0,1$ arc sec for the 7 -sided polygon (Table 10), whereas these differences are significantly larger for several faces of the 24 -sided polygon (Table 9). It is not, however, possible to find a correlation between the magnitude of these differences
and the flatness deviations of the faces given in Table 9. It must be concluded that the angles measured on the faces cannot be easily correlated with the integral quantities RMS and P-V but have a rather complex dependence on the flatness profiles of the faces.

### 6.6 Correlation analysis

The interferometric results of the flatness and pyramidality of the polygons, which were presented in Section 6.5, raise the question as to whether there are correlations between these quantities and the differences of the results of this intercomparison.

It turned out that in the case of the 7-sided polygon as well as for the 24 -sided polygon, when only 12 faces had been measured, a correlation could not be proved, perhaps also due to the poor statistics for the small number of faces. In the case of P24, with all faces measured, it was possible to find three significant correlations which are shown in the diagrams of Figures 6 to 8.

Figure 6 gives the standard deviations $s_{i}(7)$ of seven results according to Table 3, Section 6.2, with the results of NMi excluded, as a function of the RMS* values from Table 9. The correlation coefficient was found to be $r=0,70$ with an error probability $p<0,1 \%$. (When the same analysis was made with the NMi values included, the correlation is reduced to $r=0,38$.) The linear regression of the correlation in Figure 6 is described by: $s_{i}(7)=0,0323^{\prime \prime}+0,0146 " / n m \cdot R M S^{*}$. The constant offset may be attributed to measurement errors which are not influenced by the polygon faces. The RMS* values of the faces, expressing their flatness deviation from the common mean, enter with the factor $0,0146 " / \mathrm{nm}=70,8 \mathrm{nrad} / \mathrm{nm}$. The inverse of this value, which is $14,1 \mathrm{~mm}$, can be interpreted as the effective width of the face linked with this angle dependence on flatness deviation.

Another distinct correlation exists between the differences of the reduced angle deviations from the arithmetic mean of the intercomparison, when measured either using the autocollimator ELCOMAT HR or using the phase shifting interferometer, in both cases with a circular aperture limitation to about 18 mm . Figure 7 in this case shows a correlation with $r=0,76$ and $p<0,1 \%$. This proves that the limited aperture of the polygon face has a similar effect on the measured angle for both instruments, compared with the mean of the intercomparison, where no aperture limitation was used for this polygon.

The result of GUM was - by chance - the only one of this intercomparison, for which the 24 faces of P24 were measured using a circular ( 15 mm diameter) diaphragm aperture. This is the reason why the differences of the GUM values from the arithmetic mean are highly correlated with the corresponding differences of the interferometric results, as can be seen in Figure 8 ( $r=0,87 / p<0,02 \%$ ). This can be regarded as another proof for the strong influence of the aperture on the measured angle, regardless of which type of instrument or method of measurement is used.

### 6.7 Additional investigations

One of the most striking results of this intercomparison gave rise to discussions among two of the participants and the pilot laboratory, and finally led to additional investigations and results of general importance.

As mentioned in Section 6.2 and shown in Table 2, Figures 2.1-2.3, the results of NMi and the two results of CEM for the twelve faces of P24, are of a remarkable coincidence and deviation from the arithmetic mean obtained from altogether twelve results. The coincident deviation of these three results in most cases causes a common shift of the other nine results to the opposite side of the mean.

This fact was all the more astonishing as NMi and CEM had applied different methods of measurement (see Section 5), NMi using a shortened method 2 on 24 faces and CEM the complete methods 1 and 2 on twelve faces. The only similarity which could be found was due to the types of the autocollimators used: NMi applied two instruments of type RTH-DA 400 and CEM either employed instrument RTH-DA 80 or combined DA 80 and DA 400. It was finally decided with the pilot laboratory to carry out additional comparisons with these two autocollimator types on the same polygon at PTB. The instruments used for this purpose were the DA 400 from NMi and the DA 80 from PTB.

The results of these comparisons are shown in Figure 9. The polygon was measured using the two DA-autocollimators in 24 faces of full size and compared with the results of NMi and of PTB using the autocollimator Elcomat HR. All results are given as differences from the arithmetic mean of the reduced angle deviations obtained in this intercomparison. The coincidence and deviation of the results NMi, DA 80 and DA 400 compared with the Elcomat result can be clearly seen.

To test the influence of the aperture on these results, additional comparisons were made in the same setup, but using an aperture diaphragm 18 mm in diameter between the autocollimators and the polygons. The results given in Figure 10 and compared with Figure 9 reveal a strong influence of the aperture on all three autocollimators. The coincidence between DA 80 and DA 400 now vanished, the deviations of the Elcomat HR increased to the same magnitude and the total distribution exhibits a more random nature.

The DA 80/DA 400 deviations shown in Figure 9 vary between $+0,75$ " and $-0,55$ " in a very pronounced change from face to face. It is not, however, possible to find a correlation with the flatness deviations of the faces (Table 9).

## 7. Conclusions

The results of this intercomparison on two polygons of rather unequal size and flatness of their faces differed in several regards: The differences of the results from the mean, expressed as $s_{i}$ values in Tables 2, 3 and 5, as well as the range of the $s_{i}$ values for the various faces, was twice as large for the 24 -sided polygon as for the 7 sided polygon:

$$
\begin{array}{ll}
\text { P 24: } & s_{i}=0,06 " \ldots \ldots .0,32 " \\
\text { P 7: } & s_{i}=0,11^{\prime \prime} \ldots \ldots .0,16 " .
\end{array}
$$

This was apparently the consequence of the larger flatness deviation of $P$ 24, especially the roll-off close to the edges, which seems to have exerted a significant influence. A distinct correlation between the $s_{i}$ values and the RMS flatness differences of the faces from their mean could be proved for this polygon (Figure 6). In the case of the largest differences (three results of NMi and CEM), it could be shown that these were caused by the autocollimator types DA 80 and DA 400, however they were not found to be correlated with the flatness deviations of the faces.

In contrast to the differences of the results, the laboratories evaluated their uncertainties $U_{j}$ in most cases smaller for P 24 than for P 7 (see Tables 1 and 4). Mostly however, the standard deviations $\mathrm{s}_{\mathrm{j}}$ in Tables 2 and 3, expressing the deviations of each laboratory's results for all polygon faces, are not smaller than half the $U_{j}$ values, as was to be expected. The reason for this are the rather large common differences of some of the results from the arithmetic mean, shifting the
majority of the other results with their small differences to the opposite side. Fortunately, the results with the largest differences are linked with higher uncertainties. It can therefore be expected that the introduction of a weighted mean instead of the arithmetic mean as a reference, with the uncertainties entering into the weighting factors, would considerably reduce the $s_{j}$ values and thus solve the abovementioned discrepancy in comparison with the uncertainties. On the other hand, it remains to be clarified, whether unknown systematic deviations could be common to all those results, which were obtained with the same types of measuring instruments, especially as regards of the Elcomat-users OFMET, LNE, UME and PTB. Anyhow these results were also quite well confirmed by the results obtained with other autocollimators and the interferometer.

Optical aberrations and thus zero offset but not calibration errors of the autocollimators, in combination with flatness deviations of the individual polygon faces, turned out to have exerted the greatest influence on the results of this intercomparison, especially for the 24-sided polygon and the autocollimators DA 80 and DA 400. Compared with this influence, the method of measurement applied and the angular standard used appear to be of minor importance. The differences measured between the two polygon positions were astonishingly small and did not reveal the uncertainty contributions of the autocollimator/polygon combination expected.

It can finally be concluded that the evaluation of the uncertainty of angle measurement using a certain combination of polygon and autocollimator remains a difficult problem, though this intercomparison has furnished many valuable results.

## References:

[1] K. Toyoda: Report of International Comparison of Angle Standards, NRLM Japan, 1990. 12.1
[2] Guide to the Expression of Uncertainty in Measurement, BIPM et al, ed ISO, 1993
[3] R. Probst: Measurement of Angle and Flatness Deviations of Polygon Prism Faces using a Phase-Shifting Interferometer, VDI-Reports No. 1118, 1994, 173-178
[4] R. Probst, H. Kunzmann: Messung von Winkel- und Formabweichungen an Spiegelpolygonflächen mit einem Phaseninterferometer, PTB-Mitt. 103 (1993), 43-50

