FINAL PUBLISHABLE REPORT

Grant Agreement number	18NET05
Project short name	MATHMET
Project full title	Support for a European Metrology Network in mathematics and statistics

Project start date and duration:	01 June 2019, 48 months	
Coordinator: Sebastian Heidenreich, PTB Tel: +49 (0)30 3481 7726 E-mail: sebastian.heidenreich@ptb.de Project website address: https://www.euramet.org/european-metrology-networks/mathmet/		
Internal Funded Partners: 1. PTB, Germany 2. BAM, Germany 3. IMBiH, Bosnia and Herzegovina 4. INRIM, Italy 5. IPQ, Portugal 6. LGC, United Kingdom 7. LNE, France 8. NPL, United Kingdom 9. VSL, Netherlands	External Funded Partners:	Unfunded Partners:

TABLE OF CONTENTS

1 Overview

The objective of this network project was to support the associated European Metrology Network (EMN) for Mathematics and Statistics (EMN MATHMET). This was achieved by creating a Strategic Research Agenda (SRA), a stakeholder network and a Quality Management System (QMS). These activities have supported and accelerate the impact of the EMN MATHMET. This project also brought the associated EMN MATHMET to a position where it could become a point of reference for mathematics and statistics for metrology that includes input from stakeholders from industry, standardisation bodies and academia. Through this project, the associated EMN MATHMET has been supported in its promotion of best practices in mathematics and statistics for metrology.

European Metrology Network for Mathematics and Statistics (EMN MATHMET)

The precursor of the EMN MATHMET was the European Centre for Mathematics and Statistics in Metrology (MATHMET) which was established in 2014 as an outcome of the EMRP project NEW04 *Novel mathematical and statistical approaches to uncertainty evaluation*. Members from seven different EU countries joined this Centre, whose main activity was to hold a regular series of workshops and to prepare and facilitate EMPIR projects with a strong emphasis on mathematical and statistical methods. Approval for the present MATHMET Centre to become an EMN was agreed by the EURAMET General Assembly in May 2018. The MATHMET EMN was then established in June 2019 and currently has 14 NMIs, Dis and one partner that have signed the EMN Memorandum of Understanding (MoU).

2 Need

In the last decade metrology has expanded to address societal challenges related to energy, safety, climate, life sciences and health, using novel measurement modalities such as imaging, earth observation, spectroscopy and sensor networks. Model based software and mathematical algorithms have increasingly become an integrated part of measurement devices, necessitating the development of novel mathematical and statistical tools in metrology. This development includes uncertainty quantification of large sets of correlated data, handling complex systems, applications of machine learning, real-time simulations, forecasting for large-scale systems, virtual measurements and multi-physics modelling. The development of adequate mathematical and statistical tools addressing these challenges requires substantial effort that goes far beyond the capabilities of a single national metrology institute (NMI) and requires an EMN addressing these challenges. Thus in 2018 the EMN MATHMET for Mathematics and Statistics was approved by the General Assembly of EURAMET. However, in order to optimise impact, the EMN needs to strengthen its links to key stakeholders and to identify their most urgent needs, as well as to focus on mathematical support for the grand challenges of metrology and procedures for the assessment of software, data and guidelines.

3 Objectives

The overall aim of this project was to maximise and accelerate the impact of the EMN MATHMET. The project addressed the following objectives:

1. To develop an SRA for the EMN MATHMET that supports EU and EURAMET priorities by addressing the grand challenges in mathematics and statistics in metrology, e.g. large-scale and multivariate data analysis, new data analytics including machine learning, mathematical modelling, uncertainty quantification for large-scale metrology and virtual experiments and simulation.
2. To develop stakeholder consultation processes for the EMN MATHMET to enable it to identify the most urgent guidelines, software tools, virtual training and reference data in line with the SRA.
3. To create and implement a QMS that includes criteria and procedures for the assessment of advanced metrology software and guidance documents which ensures that the EMN MATHMET recommendations meet the highest quality levels and achieve wide use and substantial impact.

4 Results

This project has supported the EMN MATHMET in identifying stakeholder needs in research and quality assurance for mathematics and statistics in metrology through a stakeholder consultation process. It has supported the EMN in developing a unified European strategy and quality framework for research in mathematics and statistics in metrology. The main project results met all the project's objectives and were:

- Development and implementation of a stakeholder consultation process (Objective 2)

Then using the feedback from the stakeholder consultation process

- to develop an SRA for the EMN MATHMET (Objective 1)
- to develop a QMS (Objective 3)

4.1 To develop stakeholder consultation processes for the EMN MATHMET to enable it to identify the most urgent guidelines, software tools, virtual training and reference data in line with the SRA

The goal of the stakeholder consultation processes was to identify the most urgent stakeholder needs in the field of mathematics and statistics in metrology and to use the information for developing the SRA (Objective 1) and the QMS (Objective 3).

The stakeholder consultation processes developed in this project was implemented to become part of the infrastructure of the EMN MATHMET. It is envisaged that the stakeholder consultation processes will be sustained beyond the lifetime of this project and used by the EMN in order to help it identify and address future stakeholder needs.

The consultation process began with the mapping into a matrix of over 60 stakeholders from a wide range of fields in the metrological community, academia, industry, government, and research institutes. This mapping was done by partners PTB, NPL, VSL, BAM, IMBiH, INRIM, IPQ, LGC, LNE. This important first step ensured a broad understanding of the diverse interests, concerns, and needs across these stakeholders.

A subset of high-level priority stakeholders was then chosen (stakeholder mapping) for first contacting by partners PTB, BAM, NPL, IMBiH, INRIM, VSL, IPQ and LNE as per the project's developed communications plan (developed by partners PTB with the help of VSL, INRIM, LNE and IPQ in May 2020). Overall, three types of stakeholders were defined:

- the metrology community that interacts with EMN MATHMET partners in a service / support capacity, such as metrology organisations, EURAMET TCs, accreditation bodies, research institutes, and other EURAMET EMNs,
- the users, such as those from industry, energy, environment, and healthcare and academia, who ultimately benefit from MATHMET,
- all other users, who could benefit from MATHMET in the in the longer term.

For more efficient and successful stakeholder engagement different engagement approaches were selected for the different stakeholder types i.e. consultation, push (e.g. emails) and pull (website) communications.

Two questionnaires were developed by partners NPL, BAM, IMBiH, INRIM, IPQ, LGC, LNE and VSL for stakeholder consultations and to help integrate stakeholder needs into the SRA and QMS (Objectives 1 & 3):

1. A short questionnaire was used as preparation for stakeholder face-to-face interviews. This short questionnaire was attached to the first email sent to stakeholders together with a short description of the EMN MATHMET. The main purpose of this short questionnaire was to find out the research interests of a particular stakeholder ready for the face-to-face interviews.
2. A longer detailed questionnaire was used to define the research needs of stakeholders. This longer questionnaire could be used in two ways:

- If the stakeholder did not wish to have a face-to-face interview (or was a lower priority stakeholder), it could be used to find out as much as possible about their research needs.
- If the stakeholder is due to have a face-to-face interview, a modified version of longer questionnaire could be used as preparation for this interview. The modification of the longer questionnaire depended on the answers of the stakeholder in the short questionnaire.

A key activity in the stakeholder consultation process was the establishment of a EMN Stakeholder Advisory Committee (SAC) with high level stakeholders. This SAC was first established in 2021 and played a pivotal role in providing strategic advice, insights, and recommendations for the SRA development (Objective 1). The fist EMN SAC meeting was used to discuss urgent needs relevant for the SRA and the second SAC meeting was focussed on the draft SRA, road maps for AI and VM, and further prioritisations of urgent needs for the QMS (Objective 3), software tools, reference data, training and guidance in mathematics and statistics in metrology. By May 2023, the SAC had grown to ten members,

1. EMN Climate Observation and Ocean (COO),
2. WELMEC,
3. EUROLAB,
4. UKAS,
5. TNO,
6. the European Network for Business and Industrial Statistics (ENBIS),
7. Infineon Technologies Austria,
8. International Congress of Metrology (CFM),
9. Eurachem and
10. the Technopolis Group.

These stakeholders cover the project's research topics (Objective 1). Through SAC meetings and in-depth interviews with key stakeholders, new challenges and opportunities in mathematics and statistics in metrology were identified and explored. These activities fostered collaboration and knowledge sharing among stakeholders, enabling the SAC to represent a wide range of expert views.

The project used a joint MATHMET – ENBIS Stakeholder workshop in 2021 (Objective 1), to gain stakeholders input and to stimulated further stakeholder interest and consultation. The joint workshop was organised by INRIM with ENBIS. At the workshop the EMN and the project's QMS (Objective 3) and its application to use cases was presented to stakeholders in a special round table session in which feedback was actively sought.

Further to this the EMN hosted an international MATHMET Conference in Paris Nov 2022 (organised by LNE)) and in particular a round table session which was used to gain feedback from stakeholders on the draft SRA (Objective 1) and on how further and better engage with the EMN MATHMET. The participants included: BIPM's Joint Committee for Guides in Metrology Working Group on the Expression of Uncertainty in Measurement (JCGM- WG1:GUM), EURAMET Technical Committees for Interdisciplinary Metrology (TC-IM), Thermometry (TC-T), Photometry and Radiometry (TC-PR), Mass and Related Quantities (TC-M) and Electricity and Magnetism (TC-EM), EMN Radiation Protection, EMN Smart Electrical Grids, EMN Advanced Manufacturing, EMN-Quantum technologies, ENBIS, JCMwave GmbH, TNO, and Infineon Technologies Austria.

The final SRA (Objective 1) and QMS (Objective 3) were presented to stakeholders at the Mathematical and Statistical Methods for Metrology (MSMM) 2023 conference in Turin on May 30, 2023. This important event marked a major milestone for the project and EMN and their commitment to stakeholder engagement.

Finally, a website for the EMN MATHMET was launched in 2022 https://www.euramet.org/european-metrology-networks/mathmet/

The website includes a list of the urgent needs on software tools, guidelines, training courses and reference data in mathematics and statistics in metrology. This list was developed and prioritised with input from stakeholders at SAC meetings, the joint IMEKO-MATHMET workshop organised by IPQ in September 2022, the international MATHMET conference November 2022 at LNE in Paris.

The EMN website provides stakeholders with an information hub for mathematics and statistics in metrology, as well as case studies and details of the EMN and its current and future activities.

4.2 To develop an SRA for the EMN MATHMET that supports EU and EURAMET priorities by addressing the grand challenges in mathematics and statistics in metrology

As part of the development of the SRA, all partners (PTB, NPL, BAM, IMBiH, INRIM, IPQ, LGC, LNE, VSL) were involved in a survey of members and stakeholders of the EMN MATHMET. This survey highlighted research topics in which stakeholders have interest and NMIs and DIs can offer their expertise. The research topics were then prioritised down based on the results of surveys of members and stakeholders of the EMN MATHMET:

1. Artificial Intelligence (AI) and Machine learning (ML),
2. Computational modelling (CM) and Virtual Metrology (VM)
3. Uncertainty Quantification and Data Analysis

The first two topics are related to emerging fields in metrology driven by the digital transformation and as these research fields are relatively new in metrology there are grand challenges. The third research area is well known in mathematics and statistics in metrology and as it crosscuts the other two research topics it can be used to help address challenges in them.

Two surveys on emerging research topics AI, CM and VM within the EMN MATHMET, were also conducted and analysed. The AI survey was aligned with the AI strategy of the European Commission and key topics for the further development of AI, CM and VM in metrology and its challenges were defined from the survey.

A joint stakeholder workshop was organised between MATHMET and ENBIS in 2021, by partner INRIM. ENBIS is a platform connecting individuals and organisations, interested in theoretical developments and practical applications in the field of business and industrial statistics. As part of the ENIBS workshop a general survey was distributed to attendees (> 100). The consortium (PTB, NPL, BAM, IMBiH, INRIM, IPQ, LGC, LNE, VSL) optimised the use of this survey and extended it to include a part on the research topics for MATHMET (Objective 2). The results showed that stakeholder interest in the research topics was almost equal (44 % for Uncertainty, and 56 % for AI and Modelling and Simulation combined) and thus supported the choice of the research topics.

The project hosted an EMN SAC meeting (Objective 2) with high level stakeholders in January 2022. At the meeting the research topics were prioritised, and stakeholder needs and challenges were discussed. Further to this, four separate interviews were conducted with SAC members ENBIS, CFM, TNO and Infineon Technologies Austria on the project's QMS (Objective 3) and the research topics.

Based on the results of the above and the stakeholder consultation process (Objective 2), the project used its monthly meetings to develop a skeleton of the SRA, The skeleton SRA was structured as follows:

- Introduction
    - Scope of the SRA
    - Identifying stakeholder needs
    - Outline
- Roadmaps and recommendations
- Research Areas
    - Strategic research topic: AI and ML
    - Strategic research topic: CM and VM
    - Data analysis and Uncertainty quantification
- Summary

The monthly meetings were also used to identify the challenges and needs of the research topics and assign lead authors for each topic (in order to produce a first draft of the SRA). The key challenge for the research topics was to increase trustworthiness of algorithms and assure the quality of the results, as this is central for stakeholders and policy makers.

Strategic research topic: AI and ML

NPL lead the drafting of the research topic on AI and ML The primary focus of this part of the SRA is to address the question of how ML and deep learning (DL) can be harnessed in a principled, explainable, and transparent way to derive trusted information about physical, chemical, biological, and environmental systems from measured data. Doing so involves helping the metrology community to make good use of ML and DL without compromising established and accepted metrology principles. A secondary focus is to consider how the main concepts of metrology (such as measurement traceability, measurement uncertainty, and calibration) might be used to inform the development of standards, regulation, and policy to bring trust more generally to systems that use ML.

Within the context of these questions, the research topic is concerned with ML and DL as opposed to the much broader topic of artificial intelligence and focusses on technical aspects of the use of ML and DL as opposed to legal, ethical, and related aspects. Here, technical aspects cover mathematical and statistical issues that contribute to the trustworthiness of a prediction, including uncertainty quantification, generalisability and robustness, and interpretability, that are strongly aligned with the remit of the EMN MATHMET. However, consideration is also given to procedural issues, such as guiding the choice of ML model, the impact of the quality and provenance of training data, and the verification and validation of the ML algorithm and software used, that are also part of establishing such trustworthiness. Although currently such procedural issues are less well aligned with the remit of the EMN MATHMET, they are nevertheless important, and are issues that the EMN MATHMET may wish to consider in the future.

Strategic research topic: CM and VM

PTB lead the drafting of the research topic on CM and VM. This part of the SRA includes new developments like digital twins and virtual experiments. For this research topic, the state of the art was elaborated, and the future challenges described. To strengthen trust in computational modelling and virtual metrology the following key challenges are identified:

- Reliability and quality
- Efficiency and real-time metrology
- Uncertainty quantification.

The partners BAM, IMBiH, INRIM, IPQ, LNE, PTB, VSL reviewed the above SRA chapters and made input, particularly in the form of use cases and applications that provide instances of where the research can be expected to have impact by meeting specific needs and challenges.

As well as the research topics in the SRA, it was also important to align this with the quality criteria for data, software, algorithms and guidelines, as developed in parallel in the project's QMS (Objective 3).

The draft SRA was completed by July 2022 and distributed to the signatories of the EMN MATHMET MoU, EURAMET TCs, EMNs, EURAMET and the EMN MATHMET SAC for feedback. The draft SRA was also presented at conferences and workshops, where stakeholders were invited to provide their feedback e.g.:

- IMEKO-MATHMET 2022 Joint Symposium in Porto and IMEKO TC6 in Berlin (both Sep 2022),
- EMN COO/ EURAMET TC Virtual pavilion at the Climate Action Workshop (Sep 2022),
- The AGM of the EMN Advanced Manufacturing (Oct 2022),
- the international MATHMET conference in Paris (Nov 2022).

Based on the feedback from the above discussions plus that from the Institute of Measurement Science of the Slovak Academy of Sciences (IMS SAS), WELMEC, EUROLAB, CFM, SMD (Belgium NMI), CEM (Spanish NMI), and EURAMET's General Assembly, and from the EMN SAC, the SRA was revised to include:

- additional applications and use cases based on stakeholder needs

- a brief description of the quality assurance tools (QAT, Objective 3) and the relation to the SRA
- a link to FAIR principles
- a description of the stakeholder consultation process (Objective 2).

The final SRA was presented at the joint ENBIS-MATHMET conference in May 2023 in Turin (MSMM 2023) and was approved by Euramet's BoD in July 2023.

The SRA will provide guidance for future research in the field of mathematics and statistics in metrology. It will be available on the EMN's website and will be updated as part of the associated EMN to account for new trends and future research topics.

A key part of the SRA was not only to identify urgent challenges, but also to define activities to overcome them. To meet these research needs and challenges (as identified within the next decade), the necessary steps are defined in the roadmaps presented in Tables 1 and 2. The steps are divided into those needed in the short-term (2023–2026) and the medium-term (2027–2032). Based on the SRA and these roadmaps, the EMN MATHMET will be able to develop a work plan to address the challenges.

Table 1. Road map for the strategic research topic AI and ML.

Challenge	Short-term (2023 – 2026)	Medium-term (2027 – 2032)
Uncertainty Quantification (UQ)	• Information about general metrology requirements for UQ for ML models and based on requirements from specific application areas (advanced manufacturing, energy and environment, healthcare, food safety, etc.), information about the different sources of uncertainty and methods for quantifying the uncertainty for those sources. • Methods for UQ for ML models focussed on supervised learning (regression and classification tasks). • Methods for propagating uncertainty of test data through fixed (already trained) ML models. • Methods for propagating uncertainty of training data to quantify uncertainty of ML model parameters. • Methods for combining the effect of uncertainty from different sources: the test data, the training data, model misspecification, the training of ML models, etc. Methods to address metrology requirements for UQ: non-Gaussian distributions, correlated and heterogeneous effects, large-scale, etc.	• Methods for UQ for ML models extended to transfer learning. • Methods for UQ for ML models extended to semi-supervised and unsupervised learning (e.g., clustering). • Methods for UQ for (data-driven) ML models augmented with domain knowledge (e.g., physics-inspired neural networks [PINNs]). Information about metrology requirements for UQ for ML models in the context of reinforcement learning.
Generalisability and Robustness	• Methods and metrics to compare the predictive performance of ML models on training data and test data that account for uncertainty in such data. • Methods to assess the extent to which ML predictions are resilient to random, systematic, adversarial, and out-of-distribution perturbations in input data. • Methods to assess the sensitivity of ML predictions to new test data and its uncertainty. • Benchmarking datasets and design metrics to allow for quantitative evaluation of generalisability and robustness and comparisons of (current and new) methods.	• Methods to evaluate the global robustness of ML predictions to variations in the training data and in the hyper-parameters of the training algorithm. • Methods to improve the robustness of ML predictions, including those based on adversarial training, on modelling label error, and on automatically searching for deep neural network architectures that are inherently robust to noise and incorrectly labelled data. • Methods to assess and improve generalisability and robustness of ML models extended to transfer learning.

Challenge	Short-term (2023 – 2026)	Medium-term (2027 – 2032)
Interpretability	• Methods designed for classification tasks adapted to apply to regression tasks, and those designed for images adapted for the types of input data of relevance in metrology applications (e.g., explanation methods that are gradient-, propagation- or perturbation-based or employ surrogate models to approximate the ML model). • Benchmarking datasets and design metrics to allow for quantitative evaluation of explanations and comparisons of (current and new) explanation methods.	• Methods for the design and training of inherently interpretable networks, such as regularisation, hybrid methods and architectural adjustments. • Methods based on incorporating domain knowledge to support interpretability (e.g., physics-inspired neural networks [PINNs]).
Quality Framework	• Guidance on the specification and collection of training, validation and testing datasets to support trustworthy ML in metrology applications (e.g., addressing questions of quality, balance, bias, combining training datasets, augmenting training data, pre-processing, and cleaning). • Guidance on model choice (e.g., choice of neural network architecture, choice of kernel function for Gaussian Processes, etc.). • Guidance on model training (e.g., setting and optimising training parameters, adversarial training, etc.). • Framework to support verification and validation of ML models, including design of appropriate metrics.	• Methods for formal verification of ML models. • Framework to support reproducibility of results from data processing pipelines dependent on ML models (e.g., recognising probabilistic nature of process of model training and role of training, validation, and test data in that process). • Framework to support auditability of ML models in metrology applications. • Specification of a standard interface to support frameworks for benchmarking, validation, and certification of ML models.

Table 2: Road map for the strategic research topic Computational modelling and virtual metrology.

Reliability and Quality	• Validation framework for virtual experiments and digital twins. • Approaches to verify computational models. • Statistical procedures for the assessment of the discrepancy between standard measurements and the data from the virtual counterpart.	• Framework for key comparisons of virtual measurements devices. • Virtual test and reference standards, e.g., standardised virtual calibrations.
Efficiency and Real-Time Calculations	• Real-time methods for computationally expensive systems. • Model error determinations, e.g., surrogate models	• Methods to treat efficiently high dimensional parametric problems.
Uncertainty Quantification (UQ)	• Methods for UQ for virtual measurements. • Methods for UQ for digital twins.	• Traceability chain to a virtual/real standard. • Digital twins for metrological applications.

.

4.3 To create and implement a QMS that includes criteria and procedures for the assessment of advanced metrology software and guidance documents which ensures that the EMN MATHMET recommendations meet the highest quality levels and achieve wide use and substantial impact

A QMS was developed and implemented as additional support to the EMN MATHMET. The QMS, now renamed quality assurance tools (QAT), provides documented rules ensuring highest quality of data, software and guidelines developed by the associated EMN MATHMET.

To begin developing the QMS, NPL, with help from BAM, IMBiH, INRIM, IPQ, LGC, LNE, PTB and VSL used gap analysis to identify those areas where current practice related to guidelines, software and data quality in metrology did not meet best practice in other science domains and where that current best practice did not meet the needs of the metrology community for the research topics identified in Objective 1. Gaps were identified in the following areas:

- **General**
  - Reproducibility of computing environment (e.g. docker),
  - Scalable QM processes (i.e. light weight to full weight depending on problem)
  - Metrics to assess problem complexity
- **Data**
  - Data readiness levels
  - Quality standards for Data
  - Version control
  - Data usage risk assessment
  - Data profiling, reporting and repair
- **Software**
  - Software Certification
  - Software Traceability

Based on the gap analysis and the current best practices, a requirements document and criteria that outlined outlining the QMS was developed by NPL and agreed with partners LNE, BAM, IMBiH, INRIM, IPQ, LGC, PTB and VSL. NPL also developed templates to assist with applying the QMS requirements to data and software development projects.

The QMS takes an ISO 9001:2015 process-based approach and its overall aims are to (i) ensure MATHMET data and software is fit-for-purpose; (ii) not be over-prescriptive; (iii) require formality where essential, e.g. a common format for specifying mathematical calculations; and (iv) draw on procedures used within partners' institutions and existing QMS standards.

The last point (iv) was key and to reflect this the QMS consists of a framework (or skeleton) designed to accommodate the QMS' used within the partners' organisations (e.g. version control will be required, but no specific version control method will be mandated).

LNE, BAM, IMBiH, INRIM, IPQ, LGC, PTB, VSL and NPL selected 8 use-cases covering metrology software and metrology data to apply the above templates and obtain feedback on them. They are:

1. CASoft: Enables risks associated with decision-making in conformity assessment to be managed when measurement uncertainty is to be taken into account. https://www.lne.fr/en/software/CASoft

2. Software developed within the EMPIR project 17IND12 Met4FoF project, which performs metrological data analysis and uncertainty calculations for sensor networks and for problems involving machine learning.
3. Pressure Gauge Calibration: Used for the evaluation of measurement uncertainty for results of calibration of pressure gauge (electromechanical and mechanical manometers).
4. PyDynamic: Provides a starting point for users in metrology and related areas who deal with time-dependent, i.e. dynamic, measurements. https://pypi.org/project/PyDynamic/
5. RBD Homogeneity: For assessing reference material homogeneity data acquired via a balanced randomised block design, as described in ISO Guide 35:2017
6. LNE Uncertainty: Software to allow the evaluation of measurement uncertainty according to the framework of the GUM and its Supplement on a Monte Carlo methods.
7. **ECG database:** A large dataset of 21837 clinical electrocardiograms (ECGs) from 18885 patients. Reference: https://npluk.sharepoint.com/sites/euramet/EMN/MaS/SitePages/Home.aspx
8. **TraCIM dataset**: A reference dataset developed in the EMRP Traceability in Computationally Intensive Metrology (TraCIM) project.

These use-cases are a mixture of data and software under development and data and software for which the QMS is being applied retrospectively. The use-cases were selected to illustrate software developed for generic purposes (first, fourth and sixth) as well as that developed for particular metrology applications (second, third and fifth). All the use-cases for software relate to the research topic of “data analysis and uncertainty”. Additionally, the second use-case relates to the topic of “machine learning and big data”, and the third and fourth to the topic of “modelling, simulation and inverse problems”. The use-cases were also selected to cover different metrology applications, including advanced manufacturing (second), calibration (third), analytical chemistry (fifth) and healthcare (seventh). Finally, the use-cases were selected to maximise the involvement of partners in order to facilitate the gathering of diverse feedback to inform the development of the QMS for software and data.

LNE, VSL, IMBiH, PTB, LGC, LNE, PTB and VSL lead the use-cases and investigating using the templates developed by NPL. INRIM and NPL undertook formal reviews for a selection of them, but all partners, including BAM and IPQ, were involved in the use-case review.

IMBiH, LNE, NPL, PTB and VSL ran a workshop as part of the INRIM-organised MATHMET-ENBIS workshop (Objectives 1 & 2) and used it to gather further useful feedback on the skeleton QMS. The workshop consisted of (i) an introduction to the QMS, (ii) an overview of selected use-cases and reference data and (iii) a round table discussion where feedback on the QMS was obtained. The feedback from stakeholders included:

- It is vital that details of the QMS (e.g. templates and use-cases) are made available on the EMN MATHMET website.
- Can the QMS be used to help develop open datasets and open software and feed into relevant technical committees?
- The QMS must require any open software and open datasets that are used to be clearly acknowledged.
- The use-cases will help create trust in the QMS and they should be made available on the EMN MATHMET website.

The QMS was then updated based on this feedback.

For the Guidelines component of the QMS, a process and associated checklist were developed to help ensure a sufficient level of quality in the development, assessment, and recommendation of existing and future guidelines for mathematics and statistics in metrology. The guidelines document and checklist were distributed to project partners for review and feedback, and multiple test-cases have been identified for partners to apply the guidelines QMS and provide further feedback.

For data and software components of the QMS; PDF templates were developed, that interactively guide the user through the process of creating a quality plan for a dataset or piece of software. The templates have also

been distributed to project partners for testing and feedback and a test plan distributed to help ensure testing consistency.

Risk assessment is also key element of the QMS, as data / software development begins with a risk-analysis based quality plan. A numeric value called a Software Integrity Level (SWIL) for software or a Data Integrity Level (DIL) for data is calculated based on an assessment of complexity and criticality of the software or data product. This integrity value determines the contents of the quality plan. The integrity level is a number between 1 and 4, where 1 indicates the lowest level of risk (e.g., prototypes of software for internal use within an organisation) and 4 indicates the highest (e.g., software that is safety critical). The PDF templates help assign and review an integrity level. The integrity level is used to determine the quality management activities, i.e., the activities listed on the plan, that shall be undertaken (e.g., reviews required and whether they should be independent of the developers of the dataset or software). A linked document provides further details of the QMS and further reference material.

The QMS, now renamed quality assurance tools, is available for stakeholders to download and use from the EMN MATHMET website (here). Indeed, the EMN's QAT are already starting to be used by stakeholders and have recently seen interest from a UK medical imaging company. The company was introduced to the QAT by EMN member NPL, with whom they have been working to develop an approach for analysing calibration data. The approach is to be implemented as software and incorporated within a commercially available measuring system. The QAT was introduced to help guide the company's software development and data handling processes and to help demonstrate the quality of the software to customers and regulators. Several representatives from the company attended a webinar about the QAT hosted by NPL and VSL (more).

5 Impact

The network infrastructure developed in this project has supported the success of the associated EMN MATHMET, providing it with important basic tools for promoting best practice in mathematics and statistics for metrology. Via the outputs developed in this project the SRA (Objective 1), stakeholder consultation (Objective 2) and the QMS (Objective 3) the associated EMN MATHMET has received a kick-start in tackling the grand challenges and emerging fields in mathematics and statistics for metrology.

Through its stakeholder-inclusive approach, this project has helped to shape both short-term (2023–2026) and medium-term (2027–2032) strategic roadmaps. The process effectively identified research needs, from uncertainty quantification to efficiency in real-time calculations, aligning actions with both immediate and future challenges.

The collaboration among partners also enabled the pooling of unique skills and resources, that has not only expedited addressing current issues but also established a platform for tackling future challenges more effectively.

Direct impact

This project has involved stakeholders in an extensive consultation (Objective 2) to help identify the most urgent needs for mathematics and statistics in metrology and support the associated EMN. The project has also used its impact activities (e.g. presentations, workshops, publications, training courses to disseminate the EMN to a wider audience. Such stakeholder engagement has included:

- An international MATHMET conference was hosted by IPQ in Lisbon in Nov 2019. More than 50 participants presented and discussed contemporary methods and challenges in applications of mathematical models and statistical data analysis to measurement science.
- The first international workshop on Metrology for virtual measuring instruments VirtMet21 was jointly organised by PTB, the EMN Advanced Manufacturing and EMN MATHMET. More than 70 participants discussed simulations, virtual experiments, digital twins in metrology, quality assurance and industrial applications.
- Two training courses on measurement uncertainty were organised by PTB and INRIM in Nov 2019 and Mar 2020. The training courses were attended by more than 20 attendees from academic and scientific communities.
- In Sep 2022 a joint IMEKO-MATHMET symposium was held at which the SRA and QMS were presented to and discussed with stakeholders.
- An international MATHMET conference was hosted by LNE in Nov 2023 with more than 80 participants.
- A 2-day introductory training course for the QMS, organised by NPL and VSL was held in Mar 2023. There were approx. 50 delegates, who were provided an overview of the QMS, examples of quality plans and the opportunity to discuss their own applications. Presentations from the course have been uploaded to the BIPM e-learning platform and are now publicly available.
- In May 2023 the final SRA and QMS were presented as part of the MSMM-2023 workshop in Turin, organised by INRIM. There were 72 registered participants.

Further to this, the project has provided input to standards bodies ISO TC 158 Gas Analysis, CEN TC 264 Air quality, UNI CT 016 Quality management and statistical methods, the Eurachem Measurement Uncertainty and Traceability Working Group, ISO TC 69 Applications of statistical methods and the BIPM and CIPM JCGM WG1.

It has also provided input to EC DG CONNECT- digital single market on a White Paper on Artificial Intelligence - A European approach to excellence and trust: COM (2020) 65 final. As well as feedback to EA (European Accreditation) on a proposed revision of EA-4/02 on uncertainty evaluation.

The feedback from this project's stakeholder consultations and the above impact activities was directly used in the developed of the SRA (Objective 1) and QMS (Objective 3) and to ensure they both address stakeholder needs. In the longer term, the feedback from the stakeholder consultations can also be used by the EMN MATHMET to better target its future support for stakeholders in mathematics and statistics in metrology.

In particular, the SRA (Objective 1) will support the EMN MATHMET to direct future research activities to stakeholder needs, which in turn should increase the efficiency of the EMN in supporting necessary research developments and grand challenges at the European level

The QMS (Objective 3) developed in this project was developed to (i) ensure MATHMET data and software is fit-for-purpose, (ii) not be over-prescriptive, (iii) require formality where essential, e.g., a common format for specifying mathematical calculations, and (iv) draw on procedures used within partners' institutions and existing QMS standards. The QMS includes criteria and procedures for the assessment of advanced metrology software and guidance documents which in turn will ensure that the recommendations of the associated EMN can be based on the highest quality levels. The publication of the QMS (QAT) on the EMN website has also made it available to all stakeholders, including other institutions who can use them for improving the quality of their own work. Indeed, the EMN's QAT are already starting to be used by stakeholders and have recently seen interest from a UK medical imaging company. The company was introduced to the QAT by partner NPL, with whom they have been working to develop an approach for analysing calibration data. See https://www.euramet.org/european-metrology-networks/mathmet/bugermenu/news/details?tx_news_pi1%5Baction%5D=detail&tx_news_pi1%5Bcontroller%5D=News&tx_news_pi1%5Bnews%5D=1940&cHash=4c846dd036537a29f38e2f8f40ec8f03

Wider impact

The outcomes of this project have supported the successful start of the EMN MATHMET. This project has also contributed to the long-term goals of the EMN by (i) producing the SRA for the EMN, (ii) by developing stakeholder consultation processes for the EMN MATHMET to enable it to identify the most urgent guidelines, software tools, virtual training and reference data in line with the SRA, and (iii) finally by creating a QMS for the assessment of advanced metrology software and guidance documents which ensures that the EMN MATHMET recommendations meet the highest quality levels.

The EMN has the ability to tackle the grand research challenges of modern metrology in areas such as healthcare and medical diagnostics, industrial production and quality assessment, energy and sustainability, safety and environmental monitoring. The associated EMN MATHMET, with input from this project, will also support the development of capabilities for NMIs and stakeholders in the field of mathematics and statistics in metrology, thus leading to an improved European metrology research landscape.

6 List of publications

1. Gertjan Kok, The digital transformation and novel calibration approaches. Technisches Messen, https://doi.org/10.1515/teme-2021-0136
2. Keith Lines, Jean-Laurent Hippolyte, Indhu George, Peter Harris, A MATHMET Quality Management System for data, software and guidelines, ACTA IMEKO, https://doi.org/10.21014/actaimeko.v11i4.1348
3. Gertjan Kok, Case studies for the MATHMET Quality Management System at VSL, the Dutch National Metrology Institute, ACTA IMEKO, https://doi.org/10.21014/actaimeko.v12i2.1339

This list is also available here: https://www.euramet.org/repository/research-publications-repository-link/

7 Contact details

Markus Bär, PTB, (0) 34817688

Markus.Baer@ptb.de