EURAMET
European Association of National Metrology Institutes

Calibration of Measuring Devices for Electrical Quantities
Calibration of Oscilloscopes

EURAMET cg-7
Version 1.0 (06/2011)

Calibration Guide

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CALIBRATION OF MEASURING DEVICES FOR ELECTRICAL QUANTITIES
CALIBRATION OF OSCILLOSCOPES

Purpose

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibrations of oscilloscopes.

Authorship and Imprint

This document was developed by the EURAMET e.V., Technical Committee for Electricity and Magnetism.

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Guidance Publications

This document gives guidance on measurement practices in the specified fields of measurements. By applying the recommendations presented in this document laboratories can produce calibration results that can be recognized and accepted throughout Europe. The approaches taken are not mandatory and are for the guidance of calibration laboratories. The document has been produced as a means of promoting a consistent approach to good measurement practice leading to and supporting laboratory accreditation.

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Calibration of Oscilloscopes

1 Purpose and scope of guideline

The purpose of this guideline is to define a generally accepted procedure for the calibration of oscilloscopes. It does not cover all technical details of oscilloscopes, even if they are important for calibration. In this directive, appropriate standards and documents (see references) have been taken into account. Additionally, for a specific oscilloscope calibration, the user has to take into account the oscilloscope documentation, especially the performance verification procedure given by the manufacturer of the particular oscilloscope. Additional literature that is not directly referenced in this guideline is listed in section 5.

In this guideline, both analogue oscilloscopes (AO) and digital storage oscilloscopes (DSO) as well as sampling oscilloscopes and transient recorders (along with probes connected to them) are discussed. The calibration of voltage probes is only valid in combination with a calibrated oscilloscope where the adaption applied between both as well as the used oscilloscope channel have to be specified in the calibration certificate.

The measured values are displayed as a waveform by a cathode-ray tube (CRT) or a display, or they can be processed and stored as a digital data stream.

2 Preparation of calibration

2.1 Visual inspection, safety check, and preliminary functional checks

Prior to the calibration, the overall condition and the functionality of the oscilloscope, e.g. the functionality of control switches, knobs, and displays have to be inspected. Furthermore, the operating modes as well as the correct installation of soft- and firmware have to be checked. Software release numbers or dates have to be noted.

Defects that could inadmissibly affect the operation must be remedied before calibration.

2.2 Adjustment of reference and operating conditions

The calibration must be carried out under the reference conditions valid for the calibration facility (e.g. ambient temperature, humidity, stabilised voltage supply, harmonic distortion) and for the individual oscilloscope. If the operating conditions deviate from the reference conditions, additional measurement uncertainty contributions (see section 3.4 and chapter 7) have to be included in the uncertainty budget. During calibration, the measuring set-up must be kept in thermal equilibrium. Warm-up times specified by the manufacturer have to be obeyed.

3 Performing the calibration

3.1 Fundamental principle of oscilloscopes

Today, oscilloscopes can be categorised into three different types [1]:

a) Analogue real-time oscilloscopes offer bandwidths up to about 500 MHz. In most cases, their input impedance is switchable (e.g. between 50 Ω low impedance and 1 MΩ high impedance). The input impedance can vary with frequency, especially for high impedances. Note that manufacturers of oscilloscopes use the term “input impedance” (e.g. (50 ± 2) Ω) in a misleading way because it is valid only for DC. In most cases, the input impedance at frequencies > DC is not specified.

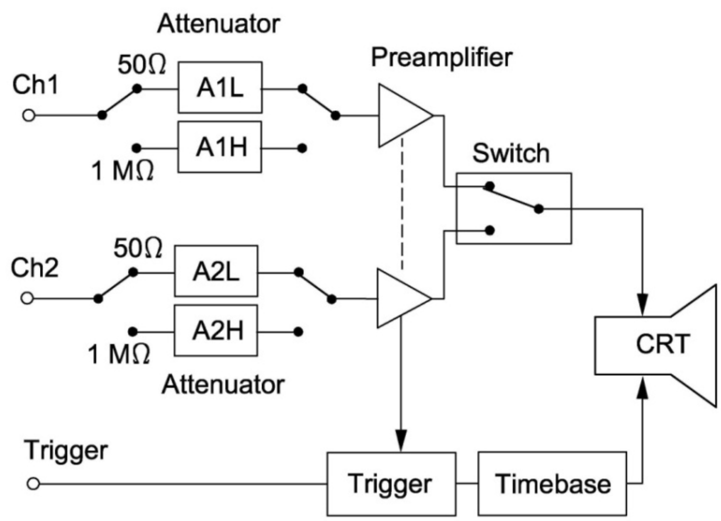

Fig 1a : Simplified block diagram of analogue real-time oscilloscope with cathode-ray tube (CRT).

b) Digital storage oscilloscopes offer bandwidths of 20 GHz and beyond. In most cases, they have a switchable input impedance, including one option matched to 50 Ω.

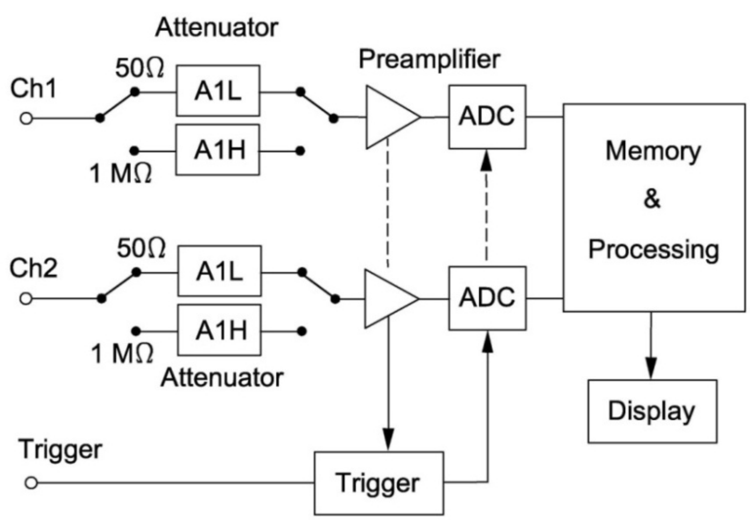

Fig 1b: Simplified block diagram of digital storage oscilloscope.

c) Sampling oscilloscopes offer bandwidths of currently up to 100 GHz. Their input connectors have an input impedance of 50 Ω, and their type is in accordance to the corresponding oscilloscope bandwidth. The voltage range of the measurement signal is limited to a few volts.

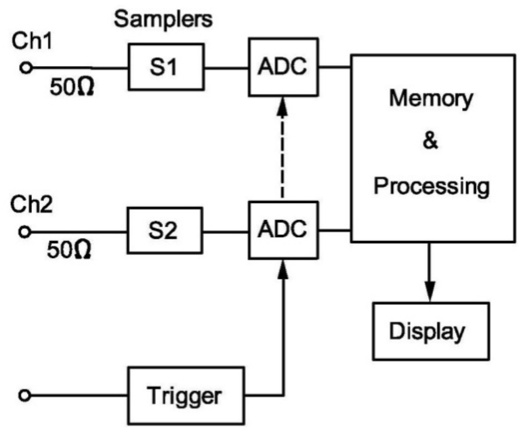

Fig. 1c: Simplified diagram of sampling oscilloscope.

3.2 Description of the device under test

Oscilloscopes are important measurement instruments to display electrical signals as waveforms. A waveform is a graphical representation of a time-dependent quantity. The oscilloscope receives an electrical signal and converts it into a waveform according to

$$y = f(x)\,. \tag{1}$$

The vertical or y-axis of the graticule typically represents an electrical voltage while the horizontal or x-axis typically represents time. The x-axis can correspond to an electrical voltage as well. An oscilloscope analyses DC voltages, AC voltages, and AC voltages having a DC offset.

Oscilloscopes consist of the following building blocks:

- vertical system
- horizontal system
- display unit
- operating unit

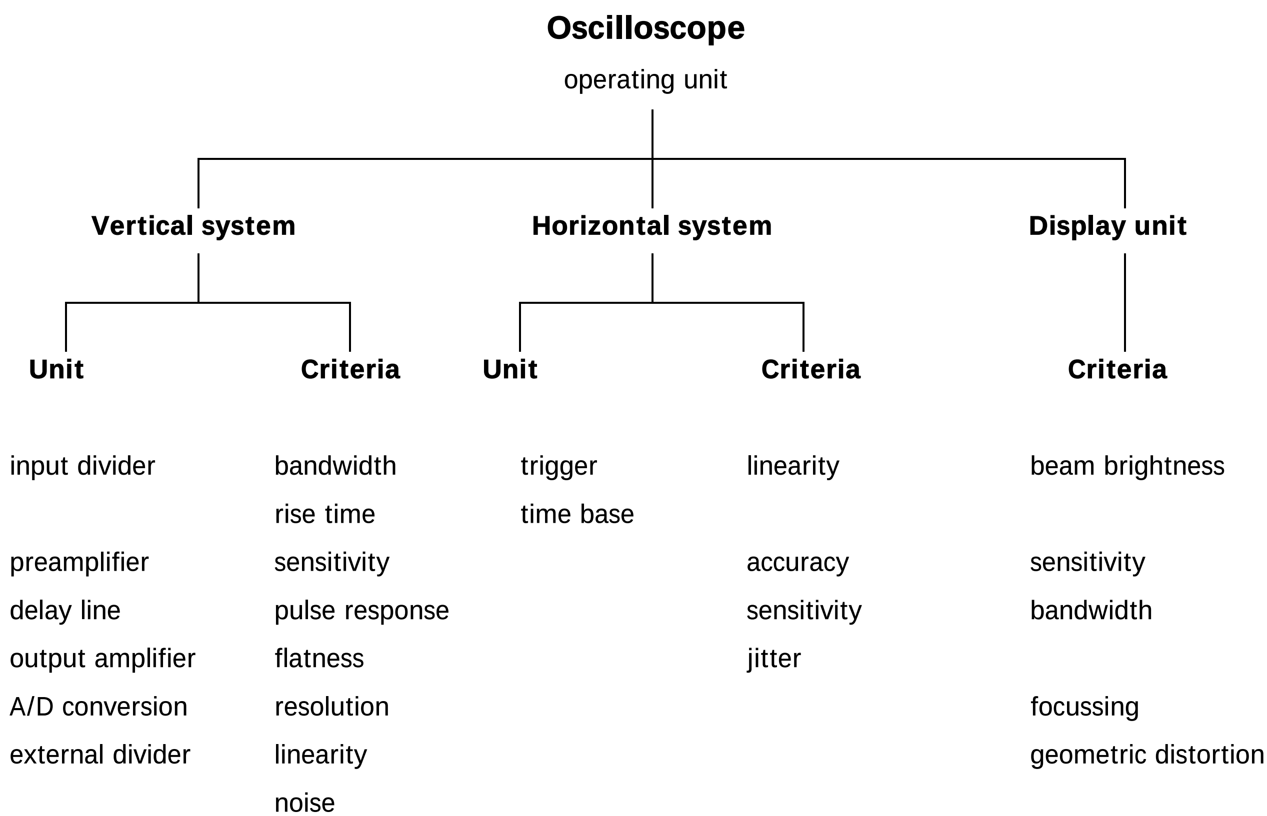

Fig. 2: Oscilloscope units and design criteria

The calibration of an oscilloscope covers the calibration of the

- vertical deflection (vertical gain)
- horizontal deflection (time base)
- rise time or alternatively bandwidth (compare section 3.3.4)
- internal calibration signals (internal references)
- trigger unit (gain and bandwidth) (optional)
- X-Y mode (gain and bandwidth) (optional)

For oscilloscopes with exchangeable plug-in units, the individual configuration (serial number of plug-in unit and plug-in position) has to be documented. The calibration is valid only for the specified configuration. The control buttons for the deflection coefficients have to be placed in a defined position which has to be documented as well.

The acquisition of the measurement data can be performed either manually or automatically (e.g. via a data interface). The following methods are applicable:

- visual reading from the display grid
- using the cursor functions
- using firmware functions (e.g. amplitude, periodic time, etc.)

The average function of digital oscilloscopes can be used to obtain a smoother signal under noisy operating conditions as for small signal to noise ratios. However, averaging has to be applied with care. Noisy operating conditions often lead to the effect of jitter. Jitter is the random scatter of the signal position with time, e.g. caused by a noisy trigger signal. Averaging of signals affected by jitter might result in a seriously distorted waveform. Evaluation of amplitude, width and rise time of an averaged waveform affected by jitter can lead to erroneous results. Possibly, this error contribution might become the dominating error of the measurement. Jitter is not further discussed in this document. For more details refer to [20].

In most cases, the acquisition method is chosen with respect to the smallest associated measurement uncertainty contribution. The applied method has to be documented.

3.3 Calibration procedures

3.3.1 Calibration of the vertical deflection (amplitude calibration)

Amplitude calibration of both the vertical and the horizontal deflection can be performed by applying the following test signals:

- DC voltage
- chopped DC voltage,
- AC voltage (sine wave),
- Pulses (e.g. LF, rectangular,).

The amplitude has to be measured in the low frequency range up to 50 kHz which can be regarded to be the linear range of the amplitude/frequency characteristic of the oscilloscope (see Fig. 9a in section 3.3.3).

In case of an AC calibration signal, a repetition frequency between 1 kHz and 100 kHz is recommended. For DSOs, the vertical position and the offset recommended by the manufacturer should be taken into account. If manufacturer's information is unavailable the amplitude reading should be performed for a 80 % coverage of the grid (Fig. 3). In contrast, for analogue oscilloscopes, a coverage of ≈ 70 % is preferable. In any case, to avoid any overload of the device under test, the operation range specified by the manufacturer has to be taken into account.

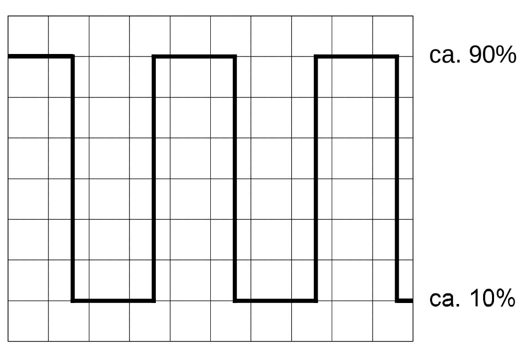

Fig. 3: Exemplary signal for calibration of vertical deflection (DSO).

The calibration has to be carried out in each voltage divider position of the high impedance input. Since the dynamic range of the A/D-converter/attenuator combination may exceed the grid, the vertical ranges of a DSO should be calibrated according to manufacturer's recommendation. If the input can be alternatively switched to a low input impedance (e.g. 50 Ω and/or has a switchable amplification (e.g. x5), the display accuracy can be measured in an arbitrary measuring range. All other measuring ranges may be calculated from the results of the high impedance measurement.

For multi-channel systems, the measurement result must be related with the corresponding oscilloscope channel. A probe calibration is only valid in combination with the calibrated oscilloscope (indicated in the calibration certificate). The probe calibration must be clearly related to the applied channel. The utilised probe must be identified in the calibration certificate.

Table 1 summarises the possible methods for traceable calibration of the vertical deflection of oscilloscopes.

Table 1: Options for traceable vertical deflection calibration.

Option	Reference standard	Working standard
1	digital voltmeter	**oscilloscope calibrator**
2	AC-calibrator	
3	**pulse generator**	

Subsequently, the options shown in Table 1 are described.

3.3.1.1 Traceability by DC

In this case, the calibration of the vertical deflection is performed with respect to a traceable DC voltage. Using a digital voltmeter or a DC calibrator as reference standard, in a first step, any kind of amplitude calibrator (e.g. oscilloscope calibrator) is calibrated (see Fig. 4). Next, the oscilloscope calibration is carried out in the chopped mode of the amplitude calibrator. An additional uncertainty contribution due to the switching from DC mode to chopped mode has to be taken into account.

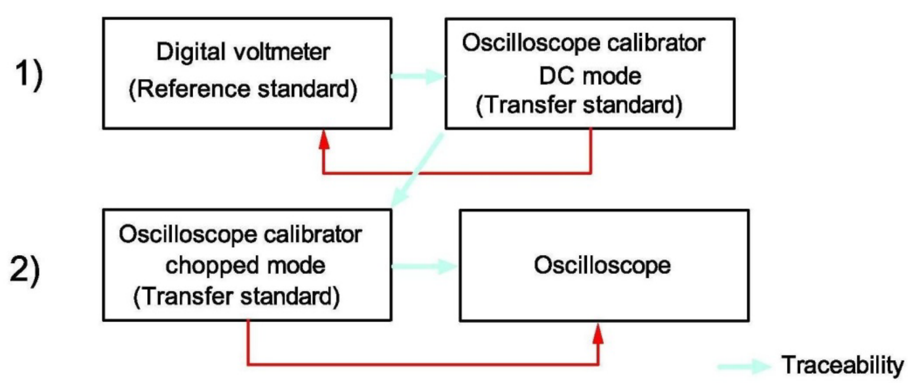

Fig. 4: Calibration of the vertical deflection, traceability by DC.

3.3.1.2 Traceability by AC

Calibration is performed by an AC voltage as measurement quantity: either an AC calibrator or a digital voltmeter is used as reference standard (Fig. 5). Since the peak value is calculated via the peak factor, it has to be ensured that a potential deviation between the measurement AC waveform and an ideal sinusoidal voltage only marginally affects the measurement uncertainty. This can be investigated by a harmonic distortion measurement.

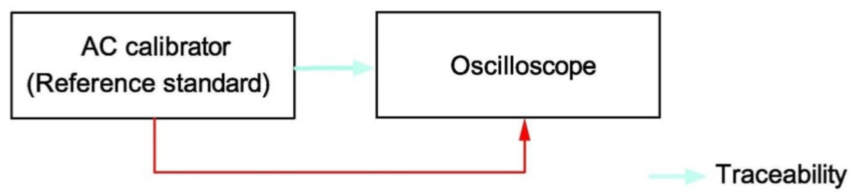

Fig. 5: Calibration of the vertical deflection, traceability by AC.

3.3.1.3 Traceability by pulse measurement

A calibrated pulse generator is used for traceable calibration of the oscilloscope (Fig. 6).

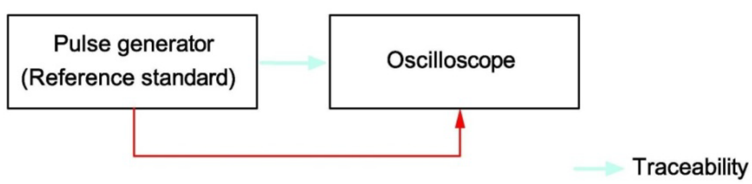

Fig. 6: Calibration of the vertical deflection: traceability by pulse measurement.

3.3.2 Calibration of the vertical deflection: traceability by pulse measurement.

In an AO, the time-base generator output voltage of the oscilloscope must be correct and linear over the entire frequency range (up to 10 decades). Thereby, only a defined point per coarse range is regarded as being calibrated. Expansion factors are commonly realised in a 1/2/5/10 or in a 1/3/10 scaling. A defined position of the horizontal fine adjustment knob (left or right stop, automatic snap-in point, preferred code for incremental adjustment) – often indicated by a LED or marked on the display – defines the nominal calibrated deflection velocity. The tuning range of the horizontal fine adjustment must enable an overlapping of the single, unequally sized sub-ranges. However, it doesn’t have to be calibrated.

The additional expansion (switching of the post-amplification) by a factor of 5 or 10 has to be calibrated. In a DSO, the accuracy of the horizontal deflection is dependent on the sampling clock accuracy and there is no sweep relation with the display. Thus, only one point is checked.

The calibration is carried out:

- for AO – in all time ranges
- for DSO – in a medium-sized time-base range

For double time base deflections, each time base has to be measured independently with the fine adjustment regulator marked as calibrated.

In general, any stable and accurate periodic signal can be used for calibration. However, pulsed signals obtained by phase-locked division or synthesis from a traceable clock rate should be preferred. Since the time base accuracy of DSOs can be very high not all oscilloscope calibrators offer the required accuracy for DSO calibration.

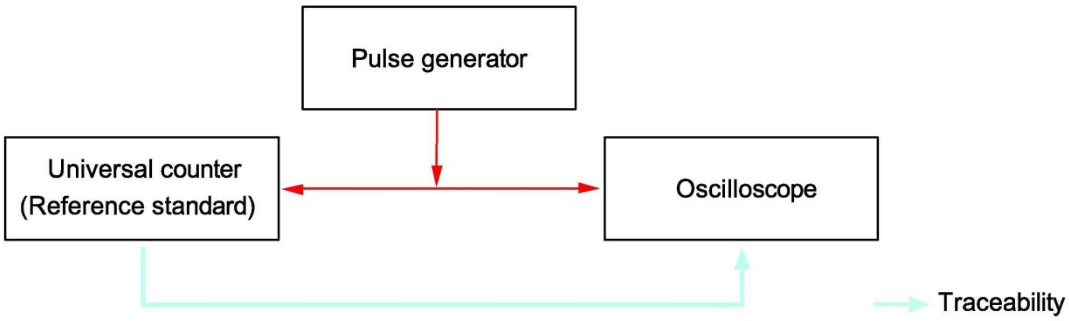

Fig. 7: Measurement and traceability of the time-base measurement.

The pulse frequency setting should result in one complete pulse period per main grid division. However, this is sometimes not applicable for high sweep rates (bandwidth of the vertical system).

According to Fig. 8, the time delay of the pulse edges (for highest rise rates) from the second (A) and from the next to the last (B) vertical main grid line has to be determined

a) in case of a fixed pulse frequency by reading/interpolation,

b) in case of a variable frequency by adjustment to coincidence with the main graticule lines and subsequent determination of the actual period. The coincidence with the first main graticule line can be adjusted by the horizontal beam controller and has to be corrected after the period adjustment.

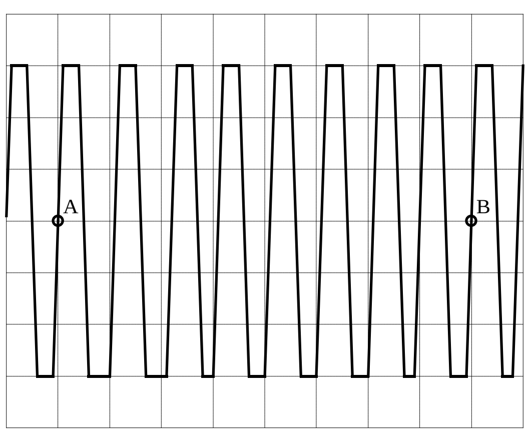

Fig. 8: Calibration of the horizontal deflection.

If a measurement function (e.g. frequency, period, pulse width) is applied it has to be specified. Operating ranges and restrictions according to the manufacturer's specifications, especially for high sweep rates, have to be taken into account. For a DSO, it may not be practical to calibrate over the grid due to lack of resolution for the accuracy of the time base. In such cases the delay function of the oscilloscope can be applied to gain resolution by measuring a longer time than the 8 divisions on the grid. Ideally, if the oscilloscope under test has an output for its frequency reference or if the chopped calibration signal output is obtained from the clock, then the frequency can be measured from the sampling clock and thus for the time base accuracy

3.3.3 Calibration of the bandwidth

The frequency point at which the amplitude response – for a constant input voltage – decreases down to 70.7 % (-3.01 dB) defines the bandwidth B. The upper frequency limit f_c is denoted as cut-off frequency (Fig. 9a).

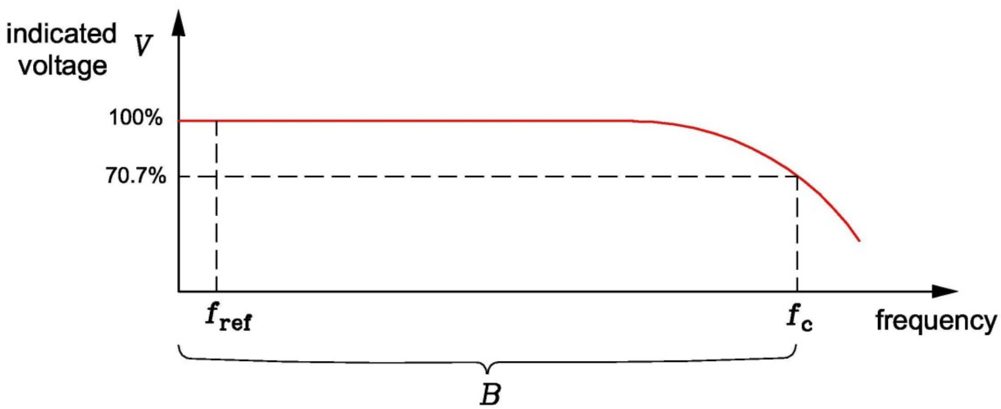

Fig. 9a: Typical frequency response of an oscilloscope.

The 3-dB point is defined by

$$-3.01\ \text{dB} = 20 \cdot \log_{10} \frac{V(f_c)}{V(f_{ref})}\bigg|_{V_{osc=const.}} , \qquad (2)$$

where $V(f_c)$ is the indicated voltage amplitude at the 3 dB-point and $V(f_{ref})$ the indicated voltage at the reference frequency f_{ref}, respectively. With respect to linear scaling, the indicated voltage drops <u>for the first time</u> below 70.7 % of the reference frequency value at the cut-off frequency.

As HF generators are generally not tuneable down to kHz frequencies, 5 % of the nominal bandwidth B is defined as the reference frequency f_{ref} (Fig. 9). In many cases, f_{ref} = 50 kHz is used. In particular, oscilloscopes having a large bandwidth show a strongly oscillating frequency response near the cut-off frequency.

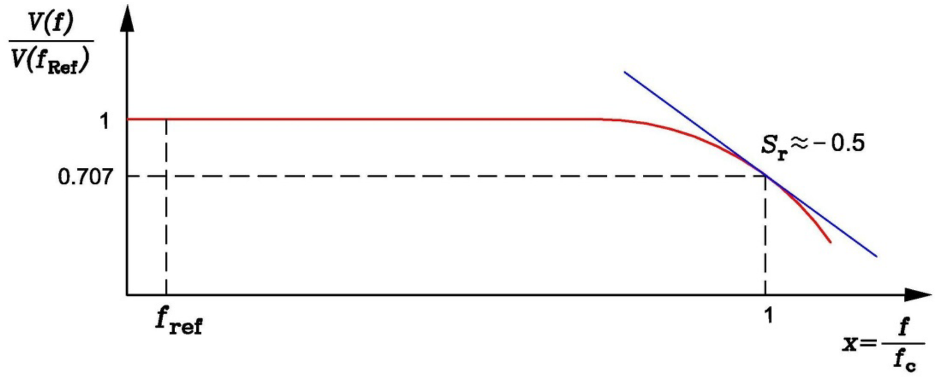

Fig. 9b: Frequency response referred to the bandwidth B for determining the slope in the 3 dB point.

When measuring the bandwidth via the 3 dB-frequency response it has to be considered that the slope of the frequency response is dependent on the oscilloscope type. For many analogue oscilloscopes, the frequency response roll-off has the characteristic of a Gaussian low pass filter. This is typical if circuit elements having a similar frequency response are cascaded. For Gaussian-low pass filters (see Fig. 9b), the relative slope S_r at the 3 dB point is given by

$$S_r = \frac{\mathrm{d}\left(\dfrac{V(f)}{V(f_{\mathrm{ref}})}\right)}{\mathrm{d}\left(\dfrac{f}{f_c}\right)} \approx -0.5$$

Thus, the uncertainty contributions of the voltage amplitude measurement to the relative measurement uncertainty of the bandwidth have to be multiplied by the sensitivity coefficient $|c_i| \approx 2$ (see Appendix section 7.2). For digital oscilloscopes, the amplitude decrease at the cut-off frequency is more significant (cp. section 3.3.4). Thus, the assumption of a Gaussian low pass filter characteristic defines the worst case. A precise determination of S_r is obtained by deviating the frequency response a few percent above and below the cut-off frequency.

3.3.3.1 Measurement of voltage amplitudes (for bandwidth determination)

For frequencies up to 100 MHz, the input impedance of oscilloscopes is typically high (typical value: 1 MΩ), whereas at higher frequencies, the input impedance is low (50 Ω in most cases). The different input impedance levels result in different voltage measurement methods [2], [3]:

a) In the high impedance range up to about 100 MHz, the voltage amplitude V_X at the oscilloscope input is measured as potential difference between the center conductor and ground – a typical method for low frequencies. In this frequency range, the source impedance is generally low compared to the load impedance.

b) In the high frequency range, however, both generator source impedance and oscilloscope input impedance are matched to the 50 Ω system of the connecting lines. In this line system, usually the amplitude of the voltage wave V_{inc}, **incident** to the oscilloscope input, is measured. V_{inc} and the input voltage amplitude V_X are related by the complex oscilloscope input reflexion coefficient Γ_X :

$$V_X = V_{inc}\left(1+\Gamma_X\right) . \qquad (4)$$

While V_{inc} slightly depends on Γ_X (see Annex 7.2), V_X significantly varies with the phase of Γ_X, especially for increasing magnitudes $|\Gamma_X|$ at higher frequencies. This variation of V_X occurs if adaptors are used resulting in a shift of the reference plane in front of the oscilloscope (see Annex A.1). Therefore, instead of V_X, the incident voltage wave V_{inc} is used as characteristic voltage of oscilloscopes for 50 Ω line systems at higher frequencies.

To avoid misinterpretation in calibration certificates, is has to be specified on which voltage (V_{inc} or V_X) the calibration is based.

3.3.3.1.1 Traceability in the high impedance range (typically below 100 MHz)

a) Calibration with voltmeter
In this frequency range, V_X can be measured with small uncertainties by using a calibrated RF-voltmeter (e.g. thermal converter). The oscilloscope under test and the voltmeter are connected in parallel via a coaxial T-junction (Fig. 10a). Thereby, the voltage measurement is indepedent of the generator source impedance. The electrical length between T-junction and both oscilloscope and voltmeter, respectively, has to be kept as short as possible, because significant deviations are caused by standing waves if the electrical length is not small compared to the wavelength. Additional adaptors can also result in increased measurement uncertainties at higher MHz frequencies and should be avoided (Fig. 10b).

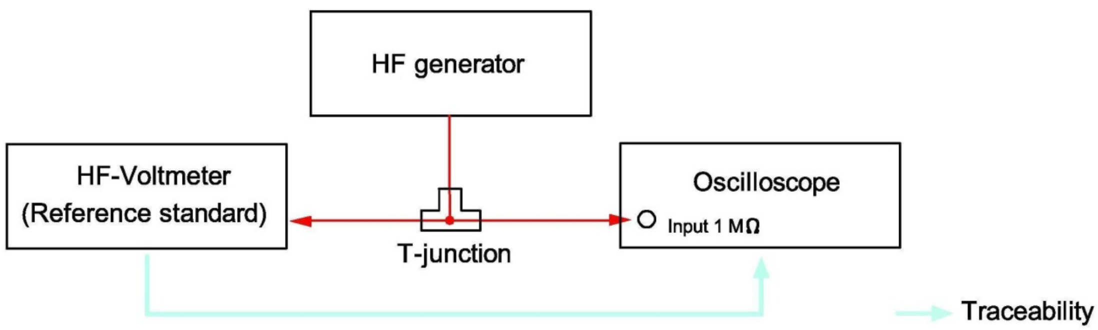

Fig. 10a: Voltage calibration of high impedance oscilloscopes by using a HF-voltmeter.

The measuring voltage is applied to the input port of the T-junction by an HF generator. The midpoint of the junction is defined as the reference plane of the voltage measurement.

Instead of a voltmeter, also a calibrated HF power meter (indication P) can be used, if its input impedance Z_{IN} is known. For an applied undistorted sinusoidal voltage the peak-to-peak voltage V_{pp} is given by

$$V_{pp} = \sqrt{8 \cdot P \cdot |Z_{in}|} \qquad (5)$$

with: P: power level indicated by the power meter

$|Z_{in}|$: magnitude of power meter input impedance (typically 50 Ω).

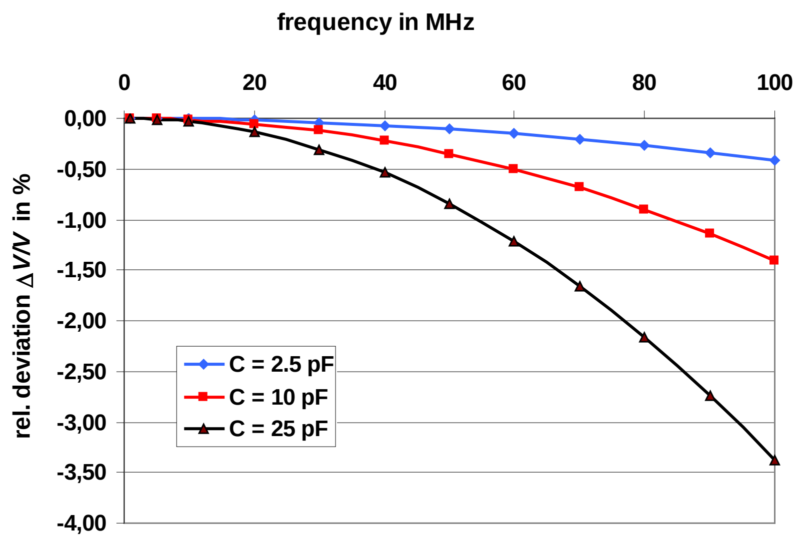

Fig. 10b: Relative voltage deviation due to a 50 Ω adapter (typical mechanical length: 2 cm) connected to a high impedance oscilloscope (input impedance 1 MΩ, parallel capacitance 2.5 pF, 10 pF, and 25 pF).

b) Calibration using an oscilloscope calibrator or a calibrated HF generator
Oscilloscope calibrators for higher frequencies generally have an output impedance of 50 Ω. Their main building-block is a frequency-tuneable HF source that delivers a stabilised, fine-adjustable sinusoidal output voltage V_{Z0} to a matched 50 Ω load impedance. Oscilloscope calibrators can also be used to calibrate high impedance oscilloscopes by inserting a 50 Ω feed-through-termination in front of the oscilloscope (Fig. 11). There are calibrators with active heads on the market that can optionally connect an internal 50 Ω resistor in parallel to their output port to perform high impedance measurements. In this case, the external feed-through termination is unnecessary.

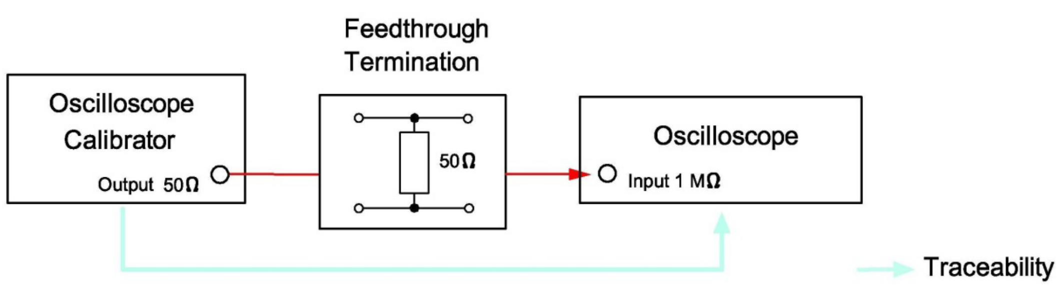

Fig. 11: Voltage calibration of a high impedance oscilloscope using a calibrator.

The indication of an oscilloscope calibrator is only valid if its output port is terminated by a matched 50 Ω load. At higher frequencies, the 50 Ω load of the feed-through termination is shunted by the oscilloscope input impedance (typically R_{in} = 1 MΩ in parallel with an input capacitance C (typical 2.5 pF to 30 pF)). This behaviour is called loading effect (see Fig. 12). The 50 Ω feed-through-termination must be directly connected to the oscilloscope input in order to minimize the input capacitance C. Instead of an oscilloscope calibrator, also a HF generator, calibrated with respect to its output voltage V_{Z0}, can be used.

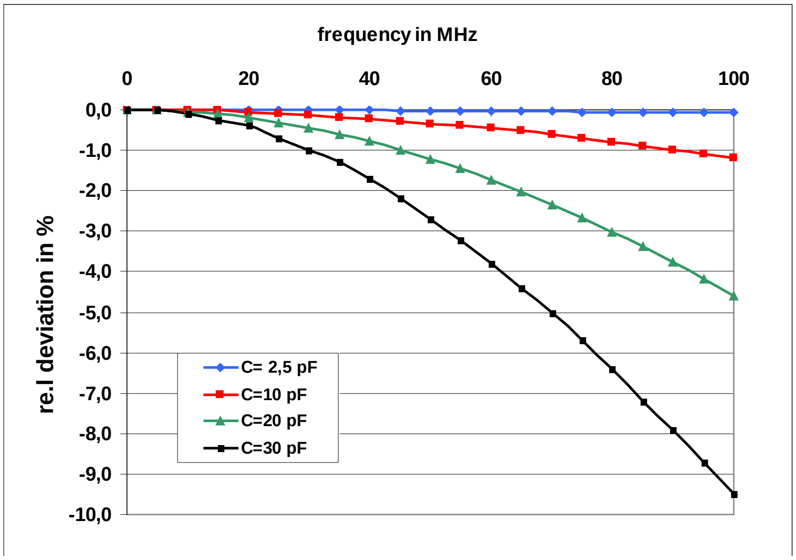

Fig. 12: Calibration of a high impedance oscilloscope (1 MΩ) using a 50 Ω-output calibrator (resp. sinusoidal HF generator) and a feed-through termination: deviation due to the oscilloscope input capacitance *C*.

3.3.3.1.2 Traceability in the 50 Ω line system

For oscilloscopes having a 50 Ω input impedance, the characteristic voltage is the voltage V_{inc} incident to the input port (cp. section 3.3.3.1). This voltage is calculated from the incident power P_{inc} according to

$$V_{inc} = \sqrt{P_{inc} \cdot Z_0}\ , \qquad (6)$$

where Z_0 denotes the characteristic impedance of the HF line system, in most cases Z_0 = 50 Ω.

a) Using a sinusoidal HF generator or power meter
A sinusoidal HF source generates the calibration signal. The power incident to the oscilloscope is determined by a symmetrical power-splitter and a calibrated power meter connected to the second port of the splitter (Fig.13). The splitter includes two (almost) identical resistors which correspond to the line impedance (50 Ω). The symmetry of a splitter can be checked by two power measurements with interchanged output ports. The measurement result can be improved by taking the average of both results. The equivalent generator system – consisting of HF-generator and power splitter - offers a good equivalent source match at the splitter output only if the power measurement at the splitter output port is referred to the reference power meter indication at other splitter output port, or if the HF generator is levelled by the reference power meter (reference standard, s. Fig.13, [11], [12], [19]). Since the equivalent output impedance of the virtual generator (HF-generator and splitter) is generally well matched to the line impedance, small measurement uncertainties can be achieved even for mismatched oscilloscopes (see Annex 7.2).

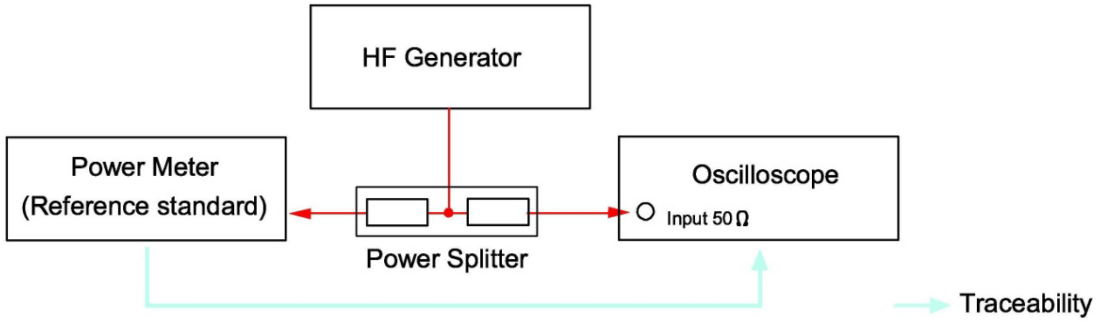

Fig. 13: Traceability by signal generator, power splitter, and calibrated power meter.

Another method for calibrating the oscilloscope utilises the same arrangement as shown in Fig.13. However, in a first step, the power indication of the reference power meter on one side of the splitter is calibrated by a power standard connected to the other side. In a second step, the oscilloscope (DUT) is connected to the splitter port instead of the power standard. Thus, it is calibrated by means of the calibrated power indication of reference power meter. This method by exchanging the DUT and the standard at the same splitter output port is often applied to calibrate power sensors by a power standard. By applying this method, the imbalance of the splitter has no influence on the measurement.

b) Using an oscilloscope calibrator
The output voltage of an oscilloscope calibrator is calibrated with respect to V_{Z0}, i.e. the calibrator output voltage applied to a matched load as illustrated in Fig. 14 (Z_0 = 50 Ω). For well matched oscilloscopes, the difference between the incident voltage V_{inc} and the indicated voltage V_{Z0} is small. Instead of a calibrator, also a HF generator can be used. However, calibrators exhibit a better source match, and furthermore, their output voltage can be adjusted in smaller increments compared to HF generators.

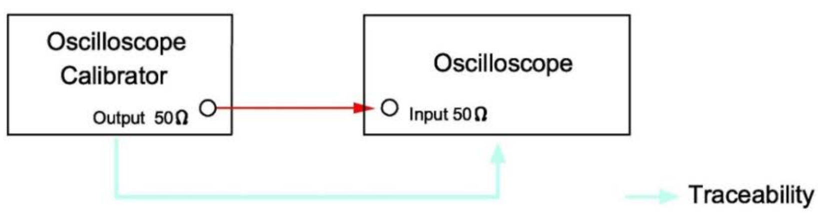

Fig. 14: Traceability by oscilloscope calibrator.

3.3.3.1.3 Influence of harmonic distortions

HF power sensors used for oscilloscope amplitude calibration measure the incident HF power which is proportional to the square of the effective value of the applied voltage. In contrast, oscilloscopes measure voltage amplitudes, i.e. peak values. To calculate amplitude values from effective values, the measurement signal must not contain harmonics of other distortions. Harmonic distortions can cause significant uncertainties since their contribution to the peak voltage is phase-dependent, whereas the contribution of harmonics to the power or voltage measured by a thermoelectric sensor is phase-independent. As an example, in the worst case, a harmonic content of -40 dBc (-30 dBc) can cause an uncertainty in the measured voltage amplitude of about 1 % (3 %).

3.3.3.1.4 Flatness

The oscilloscope bandwidth is defined as the lowest frequency at which an amplitude-invariant input signal is attenuated by 3 dB. The bandwidth as characteristic quantity does not ensure that the oscilloscope can accurately capture a HF signal. Therefore, the flatness *FV(f)* is introduced as an additional quantity which is a measure for the frequency response within the total frequency range between DC and the oscilloscope's cut-off frequency. It is defined by

$$FV(f) = \left. \frac{V(f)}{V(f_{\mathrm{ref}})} \right|_{V_{\mathrm{inc}} = \mathrm{const.}} , \tag{7}$$

where $V(f)$ denotes the indicated voltage at the measuring frequency f and $V(f_{\mathrm{ref}})$ at the (low) reference frequency, respectively, both measured for a constant input voltage. The flatness is a measure for the uniformity of the displayed signal amplitude as a function of frequency. For precise pulse amplitude measurements, it is important that the oscilloscope's flatness is constant over the entire frequency range covered by the spectral components of the pulse. To investigate the flatness, the voltage measurement procedures given above are applicable.

3.3.4 Calibration of the rise time

To perform time or pulse measurements, the oscilloscope rise time t_{r} has to be known as a characteristic quantity. It is defined as the time difference between the displayed 10 % and 90 % amplitude value (Fig. 15) if the input signal is an ideal voltage step.

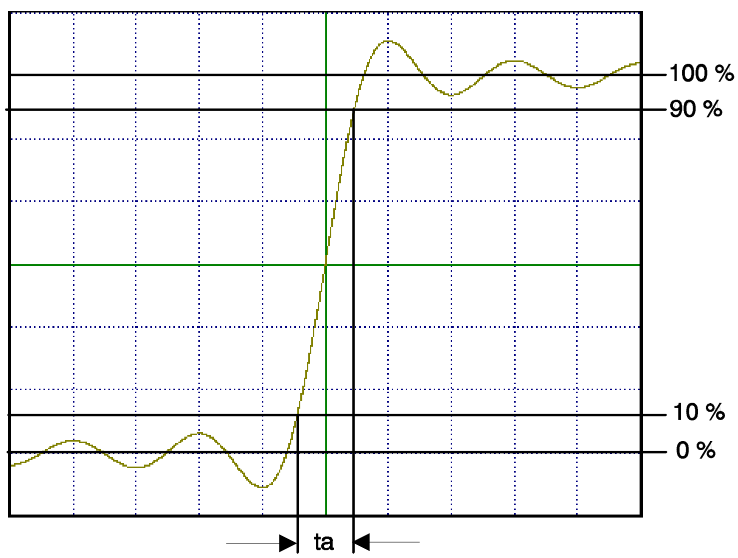

Fig 15: Definition of the rise time t_{r} .

To calibrate the oscilloscope rise time t_{r} , a non-ideal pulse with well known rise time $t_{\mathrm{r,std}}$ is generated by a calibrated pulse generator (reference standard) and applied to the oscilloscope. From the measured (indicated) rise time $t_{\mathrm{r,meas}}$, the oscilloscope rise time can be calculated by

$$t_{\mathrm{r}} = \sqrt{t_{\mathrm{r,meas}}^2 - t_{\mathrm{r,std}}^2} \ . \tag{8}$$

Eq. (8) is applicable to Gaussian-shaped characteristics of both oscilloscope and pulse. This condition can hardly be checked in the practice. For unknown characteristics, the error of eq. (8) is acceptable (< 2 %), if the condition $t_{r,meas} > 3\ t_{r,std}$ is fulfilled. For the case that $t_{r,meas} \approx t_{r,std}$, a higher effort concerning the measurement procedure is required and additional uncertainty contributions have to be taken into account. For more information see [4].

Table 2: Quantities for rise time calibration.

Symbol	Description	Remark
t_r	rise time of the oscilloscope	measurement result
$t_{r,meas}$	measured rise time	determined from displayed waveform
$t_{r,std}$	rise time of the calibrated pulse standard	given in the calibration certificate

The measurement uncertainty of the rise time is given by the uncertainties associated with the determination of the individual signal levels on the oscilloscope display, i.e. the base line value (0 % line), the pulse roof value (100 %-line, top value), and the 10 %- and 90 % marker of the pulse (Fig. 15). Jitter in connection with an activated oscilloscope average function can also have an essential influence on the measuring result. Furthermore, the uncertainty of the oscilloscope time base contributes to the overall uncertainty. Note that the pulse standard rise time $t_{r,std}$ has to be at least three times smaller than the rise time t_r of the oscilloscope [4], [5], [6]. The smaller the rise time of the pulse generator with respect to the oscilloscope rise time, the smaller is the contribution to the total uncertainty of measurement [4].

An example for calibrating the rise time t_r, a detailed measurement uncertainty budget is given in appendix 7.3.

The low-pass behaviour of analogue oscilloscope amplifiers (with specified bandwidths of less than 1 GHz) can be approximated by a Gaussian low-pass response. For such oscilloscopes, the relationship between the oscilloscope 3-dB-bandwidth B and the rise time t_r is given by

$$t_r = \frac{0.34}{B} . \qquad (9)$$

Many application notes in industry apply $t_r = 0.35 / B$ as a practical formula**.** If a calibrated pulse generator is not at hand, the oscilloscope rise time can be calculated from the measured 3dB-bandwidth. In such cases, this has to be clearly stated in the calibration certificate. For more information see [4] and [7].

3.3.5 Calibration of the trigger bandwidth and trigger sensitivity

3.3.5.1 Trigger bandwidth

Beyond the nominal or 3dB-bandwidth, the upper frequency limit of the trigger bandwidth gives information about the remaining basic functionality of the oscilloscope. Although neither the voltage nor the correct waveform can be measured correctly, at least the fundamental frequency of the test signal can be determined, along with a qualitative waveform analysis, as long as the trigger unit is operating properly. Thus, the trigger bandwidth is defined as the frequency limit

- beyond of which the trigger switch fails to respond (resulting in a partially untriggered sweep and/or in the status message "untriggered") or
- beyond of which the waveform cannot be displayed due to undersampling.

If the waveform will not be displayed any more due to excessive attenuation caused by the frequency response of the oscilloscope, the measurement is limited by the trigger sensitivity. In this case, the actual trigger bandwidth cannot be determined and only be specified either by "greater than" or, in a worst case, not at all.

3.3.5.1.1 Calibration procedure

The output of a frequency-tunable HF generator or an oscilloscope calibrator is connected to the 50 Ω input of the oscilloscope (Fig. 16).

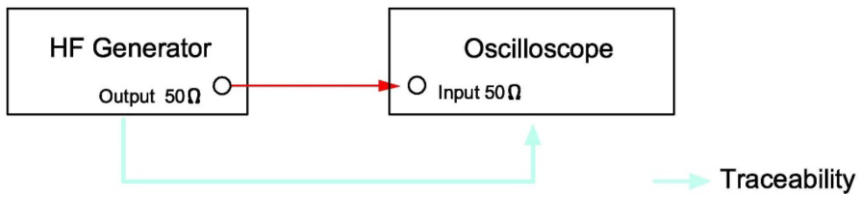

Fig. 16: Calibration of the trigger bandwidth.

The test signal with known frequency and sufficient level is displayed by the oscilloscope. It is evaluated as described above and, in case of an operating trigger unit, the frequency is determined (Fig. 17a). Next, the test signal frequency is continuously increased up to the trigger bandwidth (Fig. 17b), and the value of the latter is recorded. While increasing the frequency, the applied signal amplitude has to be adjusted by generator level tuning and oscilloscope amplifier setting to achieve approximately constant amplitude values (e.g. 5 divisions) on the display. The trigger level has to be permanently optimised to sustain signal triggering. This is performed in both the trigger "automatic" and "normal" mode.

Table 3: Configurations for calibration of the trigger bandwidth.

Configuration of the signal generator / oscilloscope calibrator	Configuration of the oscilloscope	
Effective value of voltage beginning at approx. 176.8 mV (500 mV peak-to-peak) but variable for constant amplitude on the screen Variable frequency, continuously increasing up to the trigger bandwidth limit	Amplification	Starting at 100 mV/div, variable to display 5 div
	Time axis	minimal for at highest resolution
	Trigger	AUTO or NORM
		pos. edge, starting at approx. 50% of the test signal

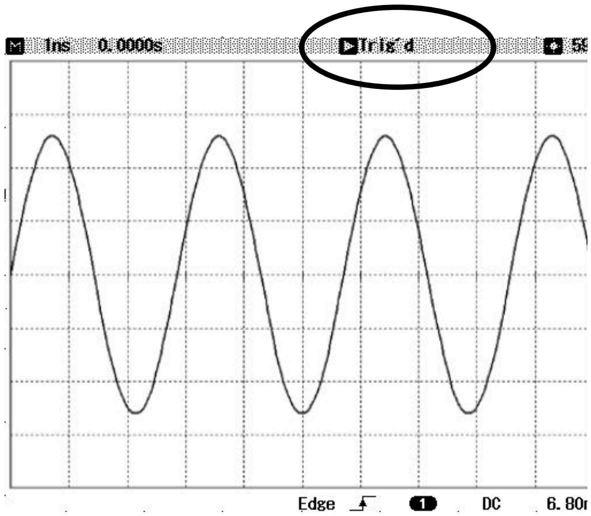

Fig. 17a: Triggered sweep near the trigger limit.

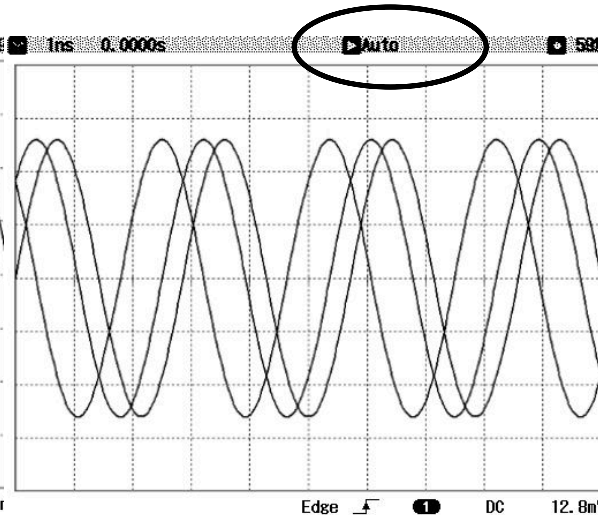

Fig 17b: Untriggered sweep.

3.3.5.2 Trigger sensitivity

The trigger sensitivity of the oscilloscope characterises the sensitivity to detect trigger events based on minimal voltage or amplitude differences required between two events. The response of the trigger circuit in case of low voltages thus enables the analysis of weak signals and their corresponding waveforms. Similar to the definition of the trigger bandwidth, the trigger sensitivity is defined as the amplitude of the input signal that is necessary to capture the input signal as a non-moving graph. Thus, the trigger circuit no longer responds for such amplitude difference (resulting in a partially untriggered sweep of the measurement curve and/or "untriggered" status message).

Typically, the nominal trigger sensitivity is given in fractions of the scale division (DIV), regardless of amplifier settings.

3.3.5.2.1 Calibration procedure

The calibration is performed in the low frequency range using an oscilloscope calibrator or an AC/DC calibrator with the output signal applied to the 1 MΩ or 50 Ω input of the oscilloscope (e.g. at 1 kHz).

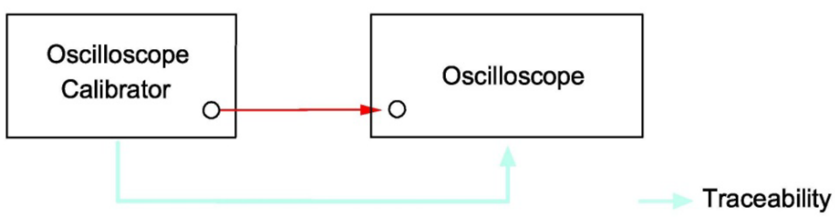

Fig. 18: Calibration of trigger sensitivity.

For a known amplitude, the test signal has to be displayed by the oscilloscope as a non-moving waveform, i.e. with the trigger circuit operating properly. Next, the generator amplitude is continuously reduced until the trigger sensitivity limit is reached. This procedure is performed both in the “automatic” and in the “normal” mode.

Table 2: Configurations for calibration of the trigger sensitivity.

Configuration on the signal generator / oscilloscope calibrator	Configuration on the oscilloscope	
Frequency = 1 kHz, sinusoidal or rectangular; Voltage amplitude decreasing beginning e.g. at 2 V, or increasing after untriggered sweep	Amplifier	1 V / div, 1 MΩ or 50 Ω
	Time axis	500 µs
	Trigger	AUTO or NORM
		pos. edge, level 0 V or zero crossing of test signal

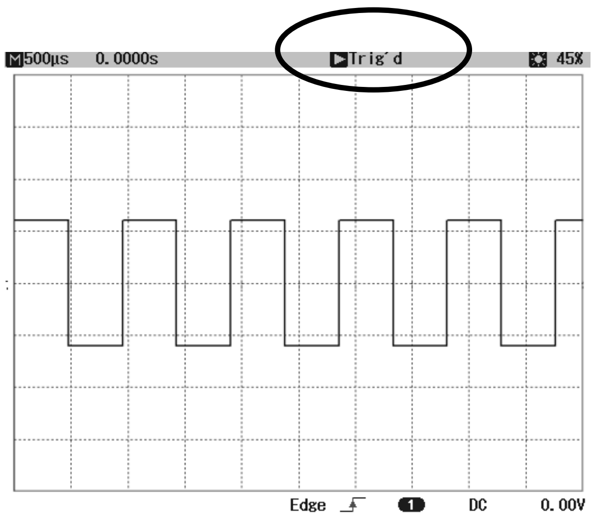

Fig 19a: Triggered sweep near the trigger sensitivity.

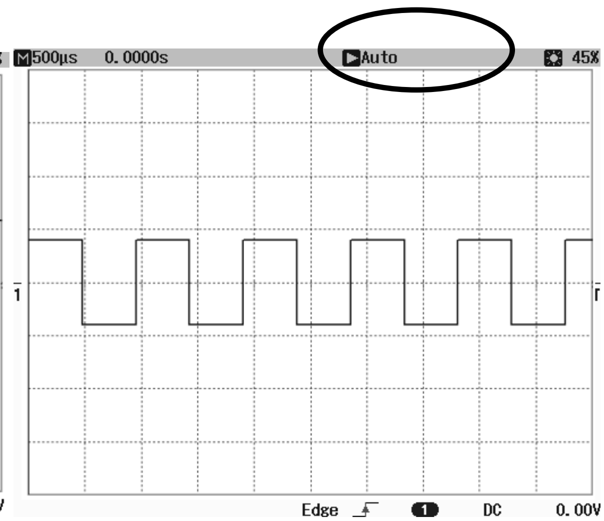

Fig 19b: Untriggered sweep, indicated by the missing status indication (e.g. Trig'd) or by a moving curve.

3.3.6 Calibration of the X-Y mode

The X-Y mode of oscilloscopes is used to measure the relation between two voltage signals, e.g. displaying I-V curves or to track phase differences (Lissajous pattern). The calibration of the X-Y mode (gain and bandwidth) can be performed with the same arrangement applied for the vertical gain and the bandwidth. Usually gain calibration will be done at one scale point (1 V/Div.), and bandwidth at 0.1 V/div. In general, the horizontal bandwidth is smaller than the vertical bandwidth.

3.3.7 Internal calibration signals

Amplitudes and frequencies of the internal calibration signals have to be calibrated in accordance with sections 3.3.1 and 3.3.2.

3.3.8 Cursor measurements

Amplitude and time measurements using a cursor are to be calibrated in accordance with section 3.3.1 and 3.3.2. Additionally, a measurement has to be performed with both cursors at the same position.

3.4 Determination of the uncertainties of measurement

The determination of contributions to the uncertainty of measurement for the measuring quantities and the total uncertainty has to be performed in accordance with accepted rules (e.g. according to the GUM [8]).

The main sources of uncertainty are:

- Standard: uncertainty of the calibrated reference standard, resolution, harmonics.
- Procedure: source impedance of the standard, input impedance of the oscilloscope, mismatch.
- Oscilloscope: uncertainty of reading, broadband noise, uncertainty of switches.

In case of DSO calibration, integrated firmware functions (averaging or interpolation resp.) can be used. Since these functions may influence the calibration result the measurement uncertainty, they have to be clearly specified in the calibration certificate.

4 Evaluation and documentation

In the calibration certificate, all measurement results for the following parameters have to be given, along with all conditions and settings relevant for the measurement:

- Amplitude
- Time deflection
- Rise time and bandwidth
- Internal calibration signals
- Cursor measurement

5 References

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6 Terms and abbreviations

A	attenuator
AC	alternating current
ADC	analogue-to-digital converter
AO	analogue oscilloscope
B	3-dB bandwidth of an oscilloscope
CRT	cathode ray tube
DC	direct current
DSO	digital storage oscilloscope
DUT	device under test
f	frequency
f_c	cut-off frequency
f_{Ref}	reference frequency
FV	flatness of the voltage characteristic
Γ	complex voltage reflection coefficient
P	HF power
P_{inc}	incident HF power
P_{ref}	reflected HF power
S_r	relative slope of a characteristic
t_r	rise time
V_{inc}	incident voltage
V_{ref}	reflected voltage
V_{Zo}	voltage at Z_0 load
V_{PP}	peak to peak voltage
V_X	LF voltage or superposition of incident and reflected voltage
Z	complex impedance
Z_0	characteristic impedance of a transmission line

7 Oscilloscope calibration: examples for uncertainty of measurement

a) The following examples give values for uncertainties that can be achieved under optimal operating conditions as they may exist at national metrology institutes. The values should by no means be considered as representative for typical everyday calibrations. They need to be evaluated and adjusted individually for each particular measurement situation.

b) The following examples do not consider the effects of noisy operating conditions. Depending on the signal amplitudes and the operating frequencies, the measured quantities might be seriously affected by noise. This will lead to additional uncertainty contributions that need to be taken into account. The effects must be estimated from repeated measurements. The averaging function of digital oscilloscopes helps to reduce the effect but it must be used carefully because averaging might lead to distorted waveforms. See the remark about averaging and jitter at the end of section 3.2

7.1 Calibration of the vertical deflection of a 100 MHz oscilloscope

7.1.1 Calibration procedure

The vertical deflection of a digital storage oscilloscope with a resolution of 10 bit is calibrated by using a calibrated multi-function calibrator with a sinusoidal output voltage. The calibration procedure is described in section 3.3.1.2. The quantity to be calibrated is the relative deviation of the vertical axis Δ_y, defined as

$$\Delta_y = \frac{V_{osc}}{V_{cal}} - 1 \ , \tag{7.1}$$

where V_{osc} denotes the oscilloscope voltage, and V_{cal} is the known calibrator output voltage that is applied to the oscilloscope. The oscilloscope settings should result in a 80% coverage of the oscilloscope graticule. From the oscilloscope display reading (number of divisions covered by the pattern) and the calibrator output voltage V_{cal} (indicated by the calibrator), the quantity Δ_y is determined.

7.1.2 Model equation

$$\Delta_{y} = \frac{N_{\mathrm{ind}} \cdot S_{\mathrm{osc}} \cdot S_{\mathrm{w}}}{2\sqrt{2} \cdot T \cdot (V_{\mathrm{cal,\,ind}} \cdot R_{\mathrm{cal}} + \delta O_{\mathrm{cal}}) + \delta V_{\mathrm{osc,noise}}} \tag{7.2}$$

With $T \approx 1$ and $R_{cal} \approx 1$, eq. (7.2) can be approximated by

$$\Delta_{y} \approx \frac{N_{\mathrm{ind}} \cdot S_{\mathrm{osc}} \cdot S_{\mathrm{w}}}{2\sqrt{2} \cdot V_{\mathrm{cal,\,ind}} \cdot R_{\mathrm{cal}} \cdot T \cdot \left(1+\frac{\delta O_{\mathrm{cal}}}{V_{\mathrm{cal,ind}}}\right)\left(1+\frac{\delta V_{\mathrm{osc,noise}}}{2\sqrt{2} \cdot V_{\mathrm{cal,ind}}}\right)} \tag{7.3}$$

where:

Δ_y	relative deviation of the vertical axis scaling
N_{ind}	indicated number of divisions on the oscilloscope display (unit: div)
S_{osc}	selected sensitivity of the oscilloscope (measuring range) (unit: V/div)
S_w	switch uncertainty of the oscilloscope
T	transmission factor due to the loading effect at the calibrator output
$V_{cal,ind}$	voltage indication of the calibrator (rms-value)
R_{cal}	readout factor of the calibrator
δO_{cal}	offset voltage of the calibrator
$\delta V_{osc,noise}$	noise voltage of oscilloscope referred to the input

7.1.3 Calculation of the uncertainties of measurement

Since a known calibrator output voltage is directly applied to calibrate the indication of the DUT (oscilloscope), the calibration can be classified as a direct measurement. To determine the measurement result, eq. (7.2) or (7.3) can be applied. They also represent the model equations for the uncertainty analysis. A multi function calibrator that generates a sinusoidal voltage of 1 kHz of known amplitude is used as reference standard. To calibrate the scale factor of the oscilloscope from 2 mV/div to 5 V /div – covering 6 divisions of the oscilloscope grid – the calibrator peak-to-peak voltage has to be adjustable between V_{pp} = 12 mV and V_{pp} = 30 V. In this example, the oscilloscope sensitivity is set to 500 mV/div.

7.1.4 Observations

The measurement procedure to determine Δ_y is repeated 12 times. The arithmetic mean value of Δ_y results in Δ_y = 0.003 3 with a standard deviation of $s(\Delta_y)$ = 0.001 2. This includes variations caused by adjusting the voltage V_{cal} to coincidence with the display grid.

7.1.5 Uncertainty contributions

N_{ind} The number of oscilloscope grids N_{ind} that are covered by the sine wave pattern of the calibrator voltage is an integer number. For digital oscilloscopes, N_{ind} = 6 is the typical value. The number is predefined and has no associated uncertainty. Deviations between the grid and the sine wave pattern – caused by a limited resolution of the display – are included in the standard deviation of Δ_y.

S_{osc} With the selected sensitivity S_{osc} of the oscilloscope (in V/div), the indicated voltage of the oscilloscope is given by $V_{osc,ind} = S_{osc} \cdot N_{ind}$. During the calibration process, $V_{osc,ind}$ is compared with the calibrator voltage $V_{cal,ind}$. The oscilloscope has a resolution of 10 bit, i.e. the 8 scales of the grid are discretised into 1024 steps. Since the calibration signal covers only 6 scales, it can be displayed with a relative resolution $\delta V/V$ of only 1/1024·6/8 = 0.073%. This relative voltage uncertainty can be considered as the relative deviation of the sensitivity: $\delta S_{osc}/S_{osc}$ = 0.00073/2. The uncertainty of δS_{osc} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

S_w For a repeated setting of the sensitivity factor, a small variation of the sensitivity was observed. In eq (7.2), this variation due to switching is taken into account by the factor $S_w = 1 + \delta S_w$ with δS_w = 0.001. The uncertainty of δS_w is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

T The oscilloscope has an input resistance of 1 MΩ, while the calibrator has an output resistance of 50 Ω. By the loading effect, the voltage at the input of the oscilloscope is reduced by about 0.005 % compared to the open circuit voltage of the calibrator. Since the resistances are given just as nominal values, the transmission factor $T = 1 \pm 0.000\,05$ considers this voltage variation. The uncertainty of $\delta T = 0.000\,05$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

$V_{cal,ind}$ The calibrator is specified with a relative deviation between output and indicated voltage of $\delta V_{cal,ind}/V_{cal,ind}$ = 0.001 5. This deviation includes a drift within 12 months as well as deviations caused by harmonics. The calibration of the oscilloscope grid is based on the peak-to-peak voltage, while the calibrators readout is given as rms-value. Thus, the factor $2 \cdot \sqrt{2}$ is included in eqs (7.2) and (7.3). The uncertainty of the deviation $\delta V_{cal,ind}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

R_{cal} The resolution factor $R_{cal} = 1 + \delta R_{cal}$ takes into account the limited resolution of the calibrator readout with δR_{CAL} = 0.000 01. The uncertainty of δR_{cal} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δO_{cal} A small offset voltage δO_{cal} is added to the nominal calibrator output voltage. The relative deviation of the calibrator voltage due to this offset amounts to $\delta O_{cal}/V_{cal,ind}$ = 0.000 2. The uncertainty contribution of this relative deviation $\delta O_{Cal}/U_{CAL,IND}$ has to be determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

$\delta V_{osc,noise}$ The voltage displayed by the oscilloscope is disturbed by a small noise voltage generated in the oscilloscope input amplifier. This amount of noise voltage with respect to the input of the oscilloscope is denoted as $\delta V_{osc,noise}$. The relative deviation of the calibration voltage caused by this offset is $\delta V_{osc,noise}/(2\sqrt{2}\ V_{cal,ind})$ = 0.000 6. The uncertainty contribution of this relative deviation has to be determined by the ‘Type B’ evaluation assuming a rectangular distribution.

7.1.6 Uncertainty budget for the relative deviation Δ_y of the vertical axis

(measurement range 500 mV/div, 6 scales of the grid are covered by the test signal)

Symbol X_i	Estimate x_i	Relative Standard uncertainty $w(x_i)$	Probability Distribution	Sensitivity Coeff.+ $\|c_i\|$	Relative uncertainty Contribution** $w_i(y)$
Δ_y	0.003 3	0.12 %	gaussian	1	0.12 %
N_{ind}	6 div	–	–	–	–
S_{osc}	0.5V/div	0.021 %	rectangular	1	0.021 %
S_w	1	0.058 %	rectangular	1	0.058 %
T	1	0.002 9 %	rectangular	1	0.002 9 %
$V_{cal,ind}$	1.057 2 V	0.087 %	rectangular	1	0.087 %
R_{cal}	1	0.000 58 %	rectangular	1	0.000 6 %
$1 + \delta O_{cal}/ V_{cal,ind}$	1	0.012 %	rectangular	1	0.012 %
$1 + \delta V_{osc,noise}/ 2\cdot\sqrt{2}\cdot V_{cal.ind}$	1	0.034 %	rectangular	1	0.034 %
Δ_y	1.003 3				0.165 %

\+ The evaluation model can be written as a product of the input quantities (eq. 7.3). Therefore, the relative standard uncertainty of measurement $w(y)$ of the measurement result can be calculated from the root of the sum of the squares of the relative standard measurement uncertainties $w(x_i)$ associated with the input values multiplied by the square of their sensitivity coefficients c_i^2. For the model eq. (7.3), these coefficients c_i equally amount to $|c_i|=1$ [11].

** The uncertainty contributions are assumed to be uncorrelated.

7.1.7 Expanded uncertainty

The relative expanded uncertainty of Δ_y associated with the measurement of the y-axis results in

$$W(\Delta_y) = k \cdot w(\Delta_y) = 2 \cdot 0.176\,\% = 0.352\,\%$$

7.1.8 Reported result

The relative deviation of the vertical axis is Δ_y = 1.0033 ± 0.003 52.

The reported expanded uncertainty of the vertical deflection is stated as the standard uncertainty of measurement multiplied by a coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

7.2 Calibration of the bandwidth of a 500 MHz-oscilloscope

7.2.1 Calibration procedure

A known HF power is incident to the 50 Ω input of the oscilloscope under test (DUT). To determine the power level, the signal is divided by a resistive power splitter and fed to a calibrated power meter (Fig. 20). The voltage $V_{osc}(f)$ incident to the oscilloscope input has a level of about 1 V (rms value, 1.4 V peak value, 2.8 V peak-to-peak value) and is calculated from the power meter indication P_{PM} according to:

$$V_{osc}(f) = \sqrt{P_{PM}(f) \cdot Z_0} \quad . \tag{7.4}$$

With this voltage held at a constant value, the generator frequency is increased until the voltage $V_{osc,ind}(f_c)$ displayed by the oscilloscope has reduced to 70.7 % referred to the value of $V_{osc,ind}(f_{ref})$ displayed at the reference frequency f_{ref} (see section 3.3.3). The cut-off frequency f_c at which the condition is fulfilled is the 3-dB bandwidth B of the oscilloscope.

To keep the input voltage constant when increasing the frequency can be difficult for a practical measurement. A more sophisticated way can be the determination of the ratio $k_{osc}(f)$ between displayed and incident voltage:

$$k_{osc}(f) = \frac{V_{Osc,ind}(f)}{V_{Osc}(f)} \tag{7.5}$$

While observing this ratio $k_{osc}(f)$, f_c is found when the following equation is fulfilled:

$$\frac{k_{osc}(f_c)}{k_{osc}(f_{ref})} = k_c = \frac{\dfrac{V_{osc,ind}(f_c)}{V_{osc}(f_c)}}{\dfrac{V_{osc,ind}(f_{ref})}{V_{osc}(f_{ref})}} = 0.7071 \tag{7.6}$$

Both procedures to determine f_c are equivalent. However, the second method can be automated easily.

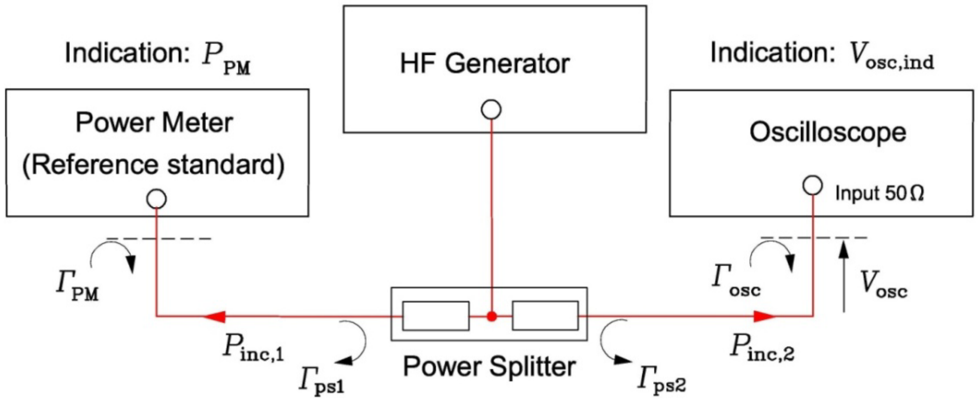

Fig. 20: Set-up for measuring the 3-dB bandwidth of the oscilloscope.

7.2.2 Model of evaluation

The measuring quantity B is the cut-off frequency (indicated by the HF generator) after frequency tuning, starting from f_{ref} . During this approach, the voltage incident to the oscilloscope has to be kept constant. As the measurement of this voltage has numerous uncertainty contributions, which also influence the uncertainty of the bandwidth B, a separate uncertainty evaluation has been performed for the voltage measurement in order to obtain a transparent uncertainty analysis. In the bandwidth model, the voltage uncertainty contributions calculated from the voltage model are multiplied by the sensitivity coefficient $1/S_r \cdot f_c / V$ (see 3.3.3).

7.2.2.1 Model of bandwidth measurement

The oscilloscope bandwidth B is determined by

$$B = f_{gen} + \delta f_{gen} + \delta f_{readout} + \frac{f_c}{S_r} \cdot \left(\frac{\delta k_c}{k_c} + \frac{\Delta k_c}{k_c} \right) \qquad (7.7)$$

with $$\frac{\delta k_c}{k_c} = \left[\frac{\delta(V_{osc}(f_c))}{V_{osc}(f_c)} + \frac{\delta(V_{osc}(f_{ref}))}{V_{osc}(f_{ref})} + \frac{\delta(V_{osc,ind}(f_c))}{V_{osc,ind}(f_c)} + \frac{\delta(V_{osc,ind}(f_{ref}))}{V_{osc,ind}(f_{ref})} \right] \qquad (7.8)$$

where:

f_{gen} frequency displayed by the HF-generator when

$V_{osc}(f) = 0.7071 \cdot V_{osc}(f_{ref})$ and with:

δf_{gen} unknown correction caused by deviations of the HF-generator frequency

$\delta f_{readout}$ unknown correction caused by the readout inaccuracy of the generator frequency

$\frac{\delta(V_{\text{osc}}(f_c))}{V_{\text{osc}}(f_c)}$	unknown correction caused by a deviation of the incident HF voltage at $f = f_c$
$\frac{\delta(V_{\text{osc}}(f_{\text{ref}}))}{V_{\text{osc}}(f_{\text{ref}})}$	unknown correction caused by a deviation of the LF voltage at $f = f_{\text{ref}}$
$\frac{\delta(V_{\text{osc,ind}}(f_c))}{V_{\text{osc,ind}}(f_c)}$	unknown correction caused by a deviation of the HF voltage indication at the osc. at $f = f_c$
$\frac{\delta(V_{\text{osc,ind}}(f_{\text{ref}}))}{V_{\text{osc,ind}}(f_{\text{ref}})}$	unknown correction caused by a deviation of the LF voltage indication at the osc. at $f = f_{\text{ref}}$
$\frac{\Delta k_c}{k_c}$	relative deviation of the voltage ratio k_c from the optimal value 0.7071
$S_r = \frac{d\left(V(f)/V(f_{\text{ref}})\right)}{d\left(f/f_c\right)}$	sensitivity coefficient of the relative voltage deviation and of the corresponding relative frequency deviation at the 3-dB point

7.2.2.2 Model of voltage measurement

At the 3 dB-point, the voltage $V_{\text{osc}}(f)$ incident to the oscilloscope is determined by:

$$V_{\text{osc}}(f) = V_{\text{PM}}(f) \cdot (1 + \delta v_{\text{gen}} + \delta v_{\text{PM}} + \delta v_{\text{resol}}) \sqrt{\frac{1}{\eta_{\text{cal}}(f) + \delta\eta_{\text{cal}}}} \cdot (1 + 0.5 \cdot \delta D) \cdot \left(1 + \delta M_{\text{PM}} + \delta M_{\text{osc}}\right) + \delta V_{\text{con}} + \delta V_{\text{ad}} + \delta V_{\text{harm}} \tag{7.9}$$

where:

$V_{\text{PM}}(f)$	voltage indication calculated from the power meter indication calculated by $V_{\text{LM}} = \sqrt{P_{\text{PM}} \cdot Z_0}$
δv_{gen}	relative correction caused by the limited resolution of the generator amplitude readout
δv_{PM}	relative correction caused by power meter nonlinearity relative correction caused by the limited resolution of the digital voltage
δv_{resol}	relative correction caused by the limited resolution of the digital voltage measurement in the oscilloscope

η_{cal}(f)	calibration factor of the power meter at frequency f
$\delta\eta_{cal}$	drift of η_{cal} since the last calibration
Z_0	nominal value of the line impedance (typical 50 Ω)
δD	deviation of the power division factor of the splitter from 1 caused by imbalance
δM_{PM}	deviation of voltage due to mismatch between power splitter and power meter
δM_{osc}	deviation of voltage due to mismatch between power splitter and oscilloscope input
δV_{con}	unknown voltage correction caused by connector instabilities
δV_{ad}	unknown voltage correction caused by a BNC-N adapter at the input of the osc.
δV_{harm}	unknown correction caused by harmonic distortions of the measuring voltage signal

7.2.3 Calculating the uncertainties of measurement

The calibration procedure is a direct measurement including two adjustments:

1) the frequency setting to fulfill $V_{osc,ind}(f_c) = 0.7071 \cdot V_{osc,ind}(f_{ref})$,
2) the oscilloscope input voltage $V_{osc}(f)$ to be kept constant versus frequency.

The two model equations to obtain the measurement result are applied as fundamental equations for the uncertainty analysis. The first standard is an adjustable HF generator, the second standard is the calibrated HF power meter to determine the incident voltage $V_{osc}(f)$. Sources of measurement uncertainty are associated with the generator signal, with the adjustment procedure, and with the determination of the incident voltage $V_{osc}(f)$. The standard uncertainties of all contributions are given in 7.2.6.1 - 7.2.6.2

7.2.4 Observations

To determine the bandwidth, the frequency adjustment is repeated three times, which results in an arithmetic mean value of B = 488 MHz. The standard deviation of the result is given in the budget. During the tuning process, the oscilloscope input voltage $V_{osc}(f)$ is kept constant at a level of 1 V. Voltage deviations at f_{ref} and at the cut-off frequency f_c are taken into account by $\delta(V_{osc}(f_{ref}))$ and $\delta(V_{osc}(f_c))$, respectively.

7.2.5 Uncertainties of measurement

7.2.5.1 Contributions of the bandwidth measurement

δf_{gen}	The generator is synchronized via DCF 77, the remaining relative frequency deviation from the displayed value f_{gen} = 488.0 MHz is less than $1 \cdot 10^{-6}$, i.e. the uncertainty contribution is δf_{gen} = 0.000 49 MHz. The uncertainty of δf_{Gen} is determined by the'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.
$\delta f_{readout}$	The tuning resolution of the signal generator is limited to 100 kHz. Thus, the frequency deviation due to the limited generator resolution is $\delta f_{readout}$ = 50 kHz. The uncertainty of $\delta f_{readout}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.
$\delta(V_{osc}(f_c))$	Its uncertainty comprises the sum of all deviations related to the incident voltage measurement at the 3-dB point. Because this contribution is calculated from the

voltage model (7.6) from several contributions (see section 7.2.6.2) its uncertainty is a standard uncertainty.

$\delta(V_{osc}(f_{ref}))$ Its uncertainty comprises the sum of all deviation related to the incident voltage measurement when the reference frequency is adjusted at the generator. Because this contribution is calculated by the voltage model from several contributions (see sections 7.2.6.2 and 7.2.6.3) its uncertainty is a standard uncertainty.

$\delta V_{osc,ind}(f_c)$ The relative resolution of the voltage reading at f_c from the graticule of the oscilloscope display is estimated to be 0.2 %. The voltage reading is $V_{osc,ind}(f_c)$ = 0.707 V, the deviation caused by the limited resolution is $\delta V_{osc,ind}(f_c)$= 0.001 4 V. The standard uncertainty of this contribution has to be determined by the 'Type B' method assuming a rectangular distribution.

Note: This example does not take noise or ADC resolution into account. Depending on the frequency, the signal to noise ratio and the type of oscilloscope these contributions will affect the voltage reading additionally and need to be taken into account. In particular readings at f_c might be affected by noise.

$\delta V_{osc,ind}(f_{ref})$ The relative resolution of the voltage reading at f_{ref} from the graticule of the oscilloscope display is estimated to be 0.2 %. The voltage reading is $V_{osc,ind}(f_{ref})$ = 1 V, the deviation caused by the limited resolution is $\delta V_{osc,ind}(f_c)$ = 0.002 0 V. The standard uncertainty of this contribution has to be determined by the 'Type B' method assuming a rectangular distribution.

See also note at $\delta V_{osc,ind}(f_c)$

Δk_c From experience it was found that the complex adjustment procedure for k_c is only possible within a deviation of Δk_c = 0.004 resulting in a relative deviation of $\Delta k_c/k_c$ = 0.006. The relative standard uncertainty of this contribution has to be determined by the 'Type B' method assuming a rectangular distribution.

S_r This sensitivity coefficient has been calculated for a Gaussian low pass filter. Its characteristic has been assumed as a worst case estimation for the filter behaviour of the DUT (cp. section 3.3.3). S_r has been calculated at the 3-dB point to $S_r(f_c)$ = 0.49. As a calculation quantity, this coefficient has no associated uncertainty.

7.2.5.2 Contributions of the voltage measurement

The values for the uncertainty contributions mentioned below are different for the two measuring frequencies f_{ref} and f_c. An example the for voltage uncertainty contribution at the cut-off frequency f_c is given in section 7.2.6.2. In section 7.2.6.3, the result for f_{ref} is summarised.

V_{PM} The power meter readout is selectable between "power mode" and "voltage mode". In the voltage mode, the indicated voltage is the incident voltage V_{PM}. The incident voltage is calculated from the measured incident power P_{PM} according to $V_{PM} = \sqrt{P_{PM} \cdot Z_0}$, Z_0 = 50 Ω is the characteristic line impedance and has no uncertainty. Hence, $\frac{\delta V_{PM}}{V_{PM}} = \frac{1}{2}\frac{\delta P_{PM}}{P_{PM}}$.

The power meter has a limited relative resolution of 0.05%. Thus, V_{PM} has a relative deviation of $\delta V_{PM}/ V_{PM}$ = 0.025 % and the absolute deviation of δV_{PM} = 0.000 3 V. The uncertainty of δV_{PM} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δV_{gen} During the bandwidth measurement, the condition for the incident voltage $V_{osc,ind}(f_c) = 0.7071 \cdot V_{osc,ind}(f_{ref})$ has to be fulfilled by adjustment of the signal generator output power. This output signal can be varied only in increments of 0.01 dB. Therefore, the condition can be fulfilled only with a deviation of δV_{gen} = 0.001 V at a voltage level of about 1 V which is a relative deviation of δv_{gen} = 0.1 %. The relative uncertainty of δv_{gen} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δV_{resol} The digital voltage measurement inside the oscilloscope cause a relative deviation δv_{resol} because of the limited resolution. For a full screen (8 scales) resolution of 10 bit (as in 7.15) the displayed voltage of the scope at f_{ref} (covering 6 scales) has a relative deviation of 0.13 % . The displayed voltage at f_c (covering only 4.2 scales) has a relative deviation of 0,19%. The relative uncertainty of δv_{resol} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δv_{PM} In general power meters are calibrated at a level between 1 mW and 10 mW. Here, the power meter is operated at 13 mW. This is taken into account by a relative deviation δv_{PM} of 1.5 %. This contribution can be more significant using a power meter with diode sensors (diode sensor are designed for low level power measurements) having larger nonlinearities. If a calibration certificate about the linearity of the power sensor is unavailable, manufacturer specifications have to be applied. The uncertainty of δv_{PM} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

η_{cal} For the power meter, the calibration factor $\eta_{cal} = P_{ind}/P_{inc}$ – the ratio between indicated and incident power – is calibrated with an expanded uncertainty $U(\eta_{cal})$. Both calibration factor and associated standard uncertainty $u(\eta_{cal})$ are given in the power meter calibration certificate and are hence directly available.

$\delta\eta_{cal}$ A potential drift of the calibration factor η_{cal} since the last calibration is taken into account by $\delta\eta_{cal}$. The uncertainty of $\delta\eta_{cal}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δD Since the balance of the power splitter is imperfect, the power $P_{inc,1}$ incident to the power meter can be related to the oscilloscope input power $P_{inc,2}$ by $P_{inc,1} = (1+ \delta D) \cdot P_{inc,2}$ where $\delta D << 1$. The uncertainty of δD is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δM_{PM} Due to the imperfect match between the power meter input port and the power splitter output port 1, a small amount of the incident power is lost by reflections. This small unknown deviation of the incident power from the calculated value is taken into account by the voltage mismatch factor $M_{PM} = 1 \pm |\Gamma_{PM}|\cdot|\Gamma_{ps1}| = 1+ \delta M_{PM}$ with the two reflection factors $|\Gamma_{PM}|,|\Gamma_{ps1}| << 1$ at the two ports. The uncertainty of δM_{PM} caused by mismatch is determined by the 'Type B' evaluation of the standard uncertainty assuming a U-shaped distribution.

δM_{osc} Due to the imperfect match between the oscilloscope input port and the power splitter output port 2, a small amount of the incident power is lost by reflections. This small unknown deviation of the incident power from the calculated value is taken into account by the voltage mismatch factor $MV_{osc} = 1 \pm |\Gamma_{osc}|\cdot|\Gamma_{ps2}| = 1+ \delta MV_{osc}$ with the two reflection factors $|\Gamma_{osc}|,|\Gamma_{ps2}| << 1$ at the two ports. The uncertainty of δMV_{osc} caused by mismatch is determined by the 'Type B' evaluation of the standard uncertainty assuming a U-shaped distribution.

δV_{con} Due to the limited connector repeatability, the losses in the input connector of the oscilloscope vary with each repeated connection.. These instabilities are considered by the contribution δV_{con}. δV_{con} increases with frequency and is experimentally determined to δV_{con} = 0.000 5 V at f_{ref} and δV_{con} = 0.001 V at $f = f_c$. The uncertainty of δV_{con} is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δV_{ad} The oscilloscope is equipped with a BNC connector at its input port. To connect the oscilloscope to the N-connector splitter, a BNC-N adapter has to be used. The voltage losses of this adapter of about 0.2 % at f_c have to be considered as an uncertainty. At f_{ref} the losses are negligible The uncertainty of δV_{ad} = 0.001 4 V is determined by the 'Type B' method assuming a rectangular distribution.

δV_{harm} The harmonic content of the HF generator signal is -45 dBc at the reference frequency f_{ref} and -40 dBc at the cut-off frequency f_c. Thus, with a level of 1 V for V_{osc}, the deviation at f_{ref} is $\delta V_{harm}(f_{ref})$ = 0.0056 V and $\delta V_{Harm}(f_c)$ = 0.01 V. The uncertainty of δV_{harm} is determined by the 'Type B' method assuming a rectangular distribution.

Note: Today oscilloscopes offer a variety of different voltage measurement modes (peak-to-peak, rms etc.). Also for sine signals, a mode for interpolation (sin(x)/x or linear) between single data points is sometimes offered (in most cases, a sin(x)/x-interpolation give better results). As already mentioned in section 3.2, the acquisition of the measurement data can influence the measurement result. Therefore the chosen modes for the voltage measurement have to be mentioned in the measurement report.

7.2.6 Uncertainty Budgets

7.2.6.1 Uncertainty budget for the bandwidth

(at a voltage level of about 1 V)

Symbol X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Probability Distribution	Sensitivity Coefficient* c_i	Uncertainty Contribution** $u_i(y)$
f_{gen}	488.0 MHz	0.50 MHz	gaussian	1	0.50 MHz
δf_{gen}	0	0.000 3 MHz	rectangular	1	0.000 3 MHz
$\delta f_{readout}$	0	0.029 MHz	rectangular	1	0.029 MHz
$\delta V_{osc}(f_c)/V_{osc}(f_c)$*	0	0.010 7	gaussian	966 MHz	10.33 MHz
$\delta V_{osc}(f_{ref})/V_{osc}(f_{ref})$*	0	0.009 3	gaussian	966 MHz	8.98 MHz
$\delta V_{osc,ind}(f_c)/V_{osc,ind}(f_c)$	0	0.002 0	rectangular	966 MHz	1.93 MHz
$V_{osc,ind}(f_{ref})/V_{osc,ind}(f_{ref})$	0	0.002 0	rectangular	966 MHz	1.93 MHz
$\Delta k_c/k_c$	0	0.003 5	rectangular	966 MHz	3.38 MHz
B	488.0 MHz				14.37 MHz

* Values calculated in sections 7.2.6.2 and 7.2.6.3.

** The uncertainty contributions are assumed to be uncorrelated.

7.2.6.2 Uncertainty budget for the incident voltage at $f = f_c$

(incident voltage V_{osc} (f_c) = 1 V at the cut-off frequency f_c = 488.0 MHz
with $|\Gamma_{PM}|$ = 0.02, $|\Gamma_{osc}|$ = 0.08, $|\Gamma_{ps1}|$ = $|\Gamma_{ps2}|$ = 0.008)

Symbol X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Probability Distribution	Sensitivity Coefficient* c_i	Uncertainty Contribution** $u_i(y)$
$V_{PM}(f_c)$	0.995 5V	0.000 20 V	rectangular	1	0.000 20 V
δV_{gen}	0	0.000 60	rectangular	1 V	0.000 60 V
δ_{resol}	0	0.001 10	rectangular	1 V	0.001 10 V
δV_{PM}	0	0.008 70	rectangular	1 V	0.008 70 V
η_{cal}	0.9910	0.002 5	gaussian	0.505 V	0.001 3 V
$\Delta\eta_{cal}$	0	0.001 2	rectangular	0.505 V	0.000 60 V
Z_0	50 Ω	–	–	–	–
δD	0	0.001 7	rectangular	0.505 V	0.000 90 V
δMV_{PM}	0	0.000 10	U-shaped	1 V	0.000 10 V
δMV_{osc}	0	0.000 50	U-shaped	1 V	0.000 50 V
δV_{con}	0 V	0.000 6 V	rectangular	1	0.000 60 V
δV_{ad}	0 V	0.000 8 V	rectangular	1	0.000 80 V
δV_{harm}	0 V	0.005 8 V	rectangular	1	0.005 8V
$V_{osc}(f_c)$	1.000 0 V				0.010 7V

** The uncertainty contributions are assumed to be uncorrelated

7.2.6.3 Uncertainty budget for the incident voltage at $f = f_{ref}$

(incident voltage V_{osc} (f_{ref}) = 1,000 V at the reference frequency f_{ref} = 1 MHz
with $|\Gamma_{PM}|$ = 0.008, $|\Gamma_{osc}|$ = 0.02, $|\Gamma_{ps1}|$ = $|\Gamma_{ps2}|$ = 0.004)

Likewise, the voltage $V_{osc}(f_{ref})$ at the reference frequency and its associated uncertainty results in $V_{osc}(f_{ref})$= 1.000 V and $u[V_{osc}(f_{ref})]$ = 0.009 3 V.

7.2.7 Expanded uncertainty

The expanded uncertainty associated with the measurement of the oscilloscope bandwidth *B* amounts to

$$U(B) = k \cdot u(B) = 2 \cdot 14.37 \text{ MHz} = 28.74 \text{ MHz}.$$

7.2.8 Reported result

The 500 MHz-Oscilloscope has a 3-dB bandwidth ***B*** = **488.0 MHz ± 28.7 MHz.**

The reported expanded uncertainty of the bandwidth is stated as the standard uncertainty of measurement multiplied by a coverage factor *k* = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

7.3 Calibration of the rise time of a 20 GHz-oscilloscope

7.3.1 Calibration procedure

In this example, an oscilloscope with a bandwidth of 20 GHz is calibrated by means of a pulse generator (pulse standard) with a known rise time. The output signal of the pulse standard is fed to the oscilloscope input, and the oscilloscope rise time is calculated from the signal indication at the oscilloscope display.

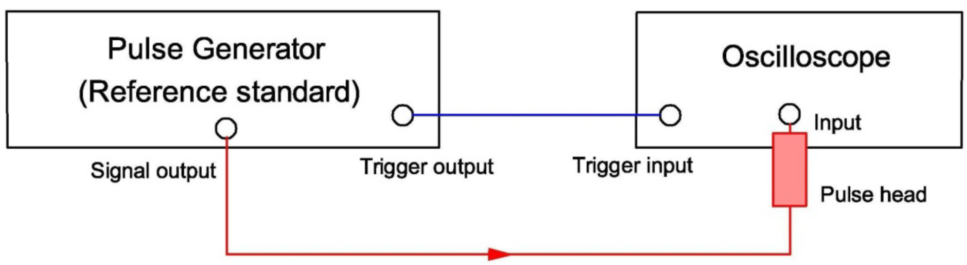

Fig. 21: Set-up for calibrating the rise time.

The measuring set-up basically consists of three components:

a) pulse generator main frame (PSPL 41016),

3) pulse head,

4) oscilloscope (DUT).

By directly connecting the external pulse head to the oscilloscope, potential distortions due to the connecting line are eliminated. To provide a clear display of the rising or the falling edge, the trigger signal of the pulse generator leads the pulse by 50 ns.

7.3.2 Model of evaluation

The rise time of the oscilloscope is determined by

$$t_{\text{r DUT}} = \sqrt{t^2_{\text{r meas}} - t^2_{\text{r standard}}}) + \delta t_{\text{method}} \qquad (7.10)$$

with:

$$t_{\mathrm{r\,meas}} = t_{90} - t_{10} + \delta t_{\mathrm{timebase}} + \delta t_{\mathrm{toplevel}} + \delta t_{\mathrm{reflevel}}, \quad (7.11)$$

where:

t_{r_DUT}	rise time of the oscilloscope under calibration (measurement result)
t_{r_meas}	measured rise time displayed by the oscilloscope
$t_{r\_standard}$	rise time of the pulse standard (given in the calibration certificate**)**
δt_{method}	correction due to the error of geometric subtraction
t_{10}	measured time at the oscilloscope at 10 % level of the full pulse voltage
t_{90}	measured time at the oscilloscope at 90 % level of the full pulse voltage
$\delta t_{timebase}$	correction due to the time base of the oscilloscope
$\delta t_{toplevel}$	correction due to the top level of displayed voltage
$\delta t_{reflevel}$	correction due to the 10 % and 90 % level of the full pulse voltage

7.3.3 Calculating the uncertainties of measurement

The calibration method is based on a direct measurement. The model equation (7.7) is also the fundamental equation for the uncertainty analysis. To calculate the measurement uncertainty the following data and uncertainty contributions are taken into account:

a) for the **pulse standard PSPL 4016,** the risetime $t_{r\_standard}$ = 6.0 ps ± 1.9 ps.

b) for the rise time measurement of the oscilloscope t_{r_meas} .

Uncertainty contributions and unknown corrections, respectively, result from the determination of the base value (0% voltage), the top value (100 % voltage), the 10 % and 90 % voltage points and from imperfections of the time base (time base accuracy $\delta t_{timebase}$).

Sometimes, it might be necessary to determine the rise time between the 20 % and 80 % voltage levels of the leading or falling edge. For a gaussian low pass, the following conversion can be applied:

$$t_{\mathrm{r}} = t_{\mathrm{r}\,10-90} = 1.5227 \cdot t_{\mathrm{r}\,20-80}$$

7.3.4 Uncertainty contributions

$t_{r\_standard}$ A pulse generator type PSPL 4016 (Picosecond Pulse Labs.) is used as pulse standard. It generates voltage pulses having an abrupt falling edge and an amplitude of about -5V. To avoid any damage of the oscilloscopes' sampling head due to the high pulse amplitude, a 20 dB attenuator is inserted between pulse head and oscilloscope input. In this configuration, the pulse standard itself has been calibrated. The rise time $t_{r\_standard}$ = 6.0 ps ± 1.9 ps (coverage factor k = 2 of the expanded uncertainty) is given in the corresponding certificate.

The pulse standard PSPL 4016 should not be used for calibration of oscilloscopes beyond 30 GHz since measurement uncertainty will increase significantly as the oscilloscope rise is shorter compared to the rise time of the standard.

t_{r_meas} The rise time t_{meas} is determined from the oscilloscope display reading. It is given by $t_{r\_meas} = t_{90} - t_{10}$. First, the 100 % level of the pulse voltage V_{100} is determined with the averaging mode switched on. Next, at the levels $0.9 \cdot V_{100}$ and $0.1 \cdot V_{100}$, the mean value of repeated measurements of t_{90} and t_{10} is determined in a 4 mV window. The difference between the two mean values gives the rise time t_{r_meas}. The uncertainties of t_{90} and t_{10} are given by the standard deviation of the repeated measurements.

t_{90} — the measured time at the oscilloscope voltage level of $0.9 \cdot V_{100}$. The standard uncertainty of t_{90} is determined by the 'Type A' method. For t_{90} = 29 ps we get: $u(t_{90)}$ = 0.2 ps.

t_{10} — the measured time at the oscilloscope voltage level of $0.1 \cdot V_{100}$. The standard uncertainty of t_{10} is determined by the 'Type A' method. For t_{10} = 10.8 ps we get: $u(t_{10)}$ = 0.2 ps.

$\delta t_{timebase}$ — unknown correction due to the time base discretisation. The estimated value of $\delta t_{timebase}$ is $\delta t_{timebase}$ = 0 ps. Only the lower and upper limits -0.4 ps and +0.4 ps, respectively, are known from the of the time base calibration. Hence, the uncertainty of $\delta t_{timebase}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

$\delta t_{toplevel}$ — unknown correction due to top level (100%) measurement of the pulse voltage. Since the pulse top is not perfectly flat, the top level voltage V_{100} is determined with a relative deviation not better than 2 %. Thus, the measured rise time t_{r_meas} has the same relative deviation of 2 %. With t_{r_meas} = 9.2 ps, the absolute deviation due to the top level measurement amounts to $\delta t_{toplevel}$ = 0.18 ps. The estimated value of $\delta t_{toplevel}$ is $E[\delta t_{toplevel}]$ = 0 ps. Only the lower and upper limits –0.18 ps and +0.18 ps, respectively, are known. Thus, the uncertainty of $\delta t_{toplevel}$ is determined by the 'Type B' method assuming a rectangular distribution.

$\delta t_{reflevel}$ — unknown correction due to deviations of the 90 % and 10 % voltage level. The determination of the points t_{90} and t_{10} – corresponding to the voltage levels V_{90} and V_{10} – is associated with uncertainty contributions which are estimated to be 0.5 % of V_{100}. As the slope of the v-t characteristic is given by $\Delta u/\Delta t$ =0.8·0.5V/9.2 ps, the deviations δt_{90} and δt_{10} of the measured times t_{90} and t_{10} result in 0.06 ps each. As these deviations are uncorrelated, the overall deviation is $\delta t_{reflevel}$ = 0.08 ps. The estimated value of $\delta t_{reflevel}$ is $E[\delta t_{refleve/}]$ = 0 ps with a maximal deviation of +0.08 ps and –0.08 ps, respectively. Because only the limits of the deviation are known, the uncertainty caused by $\delta t_{reflevel}$ is determined by the 'Type B' method assuming a rectangular distribution.

δt_{method} — unknown correction due to simplified geometric subtraction of measured rise time and rise time of the pulse standard. The estimated value of δt_{method} is $E[\delta t_{refleve/}]$ = 0 ps with a maximal deviation of ± 0.37 ps (2 % of 18.5 ps). Because only the limits of the deviation are known, the uncertainty caused by δt_{method} is determined by the 'Type B' method assuming a rectangular distribution.

Since the measurement was carried out in the statistic mode, the jitter doesn't have any influence on the measurement result (the statistic mode is a display mode without averaging, but a superposition of a number of graphs in one comprehensive view, from the thickness of the resulting graph the intensity of the jitter can recognized). The influence of jitter to rise time measurement is discussed in [20].

7.3.5 Uncertainty budget (rise time $t_{r\,DUT}$ of the 50 GHz oscilloscope)

Symbol X_i	Estimate x_i	Standard uncer-tainty $u(x_i)$	Probability Distribution	Sensitivity Coefficient* c_i	Uncertainty Contribution** $u_i(y)$
$t_{standard}$	6.00 ps	0.95 ps	gaussian	0.86	0.82 ps
t_{90}	29.30 ps	0.20 ps	rectangular	1.3	0.26 ps
t_{10}	10.80 ps	0.20 ps	rectangular	1.3	0.26 ps
δt_{method}	0.0 ps	0.21 ps	rectangular	1	0.21 ps
$\delta t_{timebase}$	0.0 ps	0.23 ps	rectangular	1.3	0.30 ps
$\delta t_{toplevel}$	0.0 ps	0.09 ps	rectangular	1.3	0.12ps
$\delta t_{reflevel}$	0.0 ps	0.05 ps	rectangular	1.3	0.07 ps
$t_{r\,DUT}$	17.5 ps				0.98 ps

* The sensitivity coefficients c_i are determined as the partial derivatives of the model of evaluation (7.11) with respect to the input quantity x_i .

** The uncertainty contributions are assumed to be uncorrelated.

7.3.6 Expanded uncertainty

The expanded uncertainty associated with the measurement of the oscilloscope rise time t_{r_DUT} is

$$U(t_{r\,DUT}) = k \cdot u(t_{r\,DUT}) = 2 \cdot 0.96 \text{ ps} = 1.9 \text{ ps} .$$

7.3.7 Reported result

The 20 GHz oscilloscope has a rise time of t_{r_DUT} = 17.5 ps ± 2.0 ps.

The reported expanded uncertainty of the rise time is stated as the standard uncertainty of measurement multiplied by a coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

7.4 Calibration of the trigger bandwidth

7.4.1 Calibration procedure

This example analyses the uncertainty of the trigger bandwidth measurement of a 500 MHz oscilloscope. As described in section 3.3.5.1, a RF signal generator with adjustable frequency is used.

7.4.2 Model of evaluation

The model equation for the trigger bandwidth is given by

$$f_{\text{trigger}} = f_{\text{generator}} + \delta f_{\text{generator}} + \delta f_{\text{v}} + \delta f_{\text{hysteresis}} + \delta f_{\text{readout}} \quad , \qquad (7.12)$$

where:

f_{trigger}	trigger cut-off frequency to be calculated
$f_{\text{generator}}$	frequency displayed by the generator when trigger function starts to fail
$\delta f_{\text{generator}}$	unknown correction caused by deviations of the generator frequency
δf_V	unknown correction determined by generator voltage fluctuation
$\delta f_{\text{hysteresis}}$	unknown correction caused by the measurement repeatability and hysteresis
$\delta f_{\text{readout}}$	unknown correction caused by the readout inaccuracy

7.4.3 Calculating the uncertainties of measurement

The calibration procedure is a direct measurement. The model equation applied to calculate the measurement result is also the fundamental equation of the model for the uncertainty analysis. The standard is an adjustable HF signal generator. The main measurement uncertainty contributions are the signal frequency uncertainty and the deviations between the frequency tuned at the generator and the true value of f_{Trigger} given by definition.

7.4.4 Observations

The trigger frequency measurement was repeated ten times. In the ascending mode, the trigger frequency measurement gave a constant value of f_{trigger} = 633 MHz. In the descending mode, also a constant value of f_{trigger} = 632 MHz was measured. The measurement invariance is due to the limited resolution of 1 MHz of the signal generator. The mean value 632.5 MHz was taken as measurement result.

7.4.5 Contributions to the uncertainty of measurement

$\delta f_{\text{generator}}$ The generator is synchronized via DCF77. The remaining maximum relative frequency deviation from the displayed value $f_{\text{generator}}$ is $1{\cdot}10^{-6}$ and the uncertainty contribution is $\delta f_{\text{generator}}$ = 0.000 63 MHz. As only the lower and upper limits of this contribution are known, the uncertainty of $\delta f_{\text{Generator}}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

δf_V A non-ideal signal voltage amplitude as well as nonlinearities of the input amplifier may affect the trigger. However, the measurement revealed that voltage amplitude variations near the trigger cut-off frequency have no significant effect on the trigger performance.

$\delta f_{\text{hysteresis}}$ For repeated measurements, a hysteresis interval of ±0.5 MHz has been observed. As only the lower and upper limit of this contribution is known, the uncertainty of $\delta f_{\text{hysteresis}}$ is determined by the 'Type B' evaluation of the standard uncertainty assuming a rectangular distribution.

$\delta f_{\text{readout}}$ The tuning resolution of the signal generator is limited to 1 MHz. Hence, the uncertainty contribution caused by the readout is 0.5 MHz. The standard uncertainty caused by $\delta f_{\text{Readout}}$ is determined by the 'Type B' method assuming a rectangular distribution.

7.4.6 Uncertainty budget (trigger bandwidth $f_{Trigger}$ of a 500 MHz-oscilloscope)

Symbol X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Probability Distribution	Sensitivity coefficient $\|c_i\|$ *	Uncertainty contribution $u_i(y_i)$**
$f_{trigger}$	632.5 MHz	0	gaussian	1	0
$\delta f_{generator}$	0	0.0003 MHz	gaussian	1	0.000 3 MHz
δf_v	0	–	rectangular	1	–
$\delta f_{hysteresis}$	0	0.29 MHz	rectangular	1	0.29 MHz
$\delta f_{readout}$	0	0.29 MHz	rectangular	1	0.29 MHz
$f_{Trigger}$	632.5 MHz				0.41MHz

* The sensitivity coefficients c_i are determined as the partial derivative of the model of evaluation (7.9) with respect to the input quantity x_i

** The uncertainty contributions are assumed to be uncorrelated.

7.4.7 Expanded uncertainty

The expanded uncertainty associated with the measurement of the oscilloscope trigger bandwidth $f_{trigger}$ is:

$$U(f_{trigger}) = k \cdot u(f_{trigger}) = 2 \cdot 0.65\,\text{MHz} = 0.82\,\text{MHz}\,.$$

7.4.8 Reported result

The 500 MHz-oscilloscope has a trigger bandwidth of $f_{trigger}$ = 632.5 MHz ± 0.8 MHz.

The reported expanded uncertainty of the trigger bandwidth is stated as the standard uncertainty of measurement multiplied by a coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.